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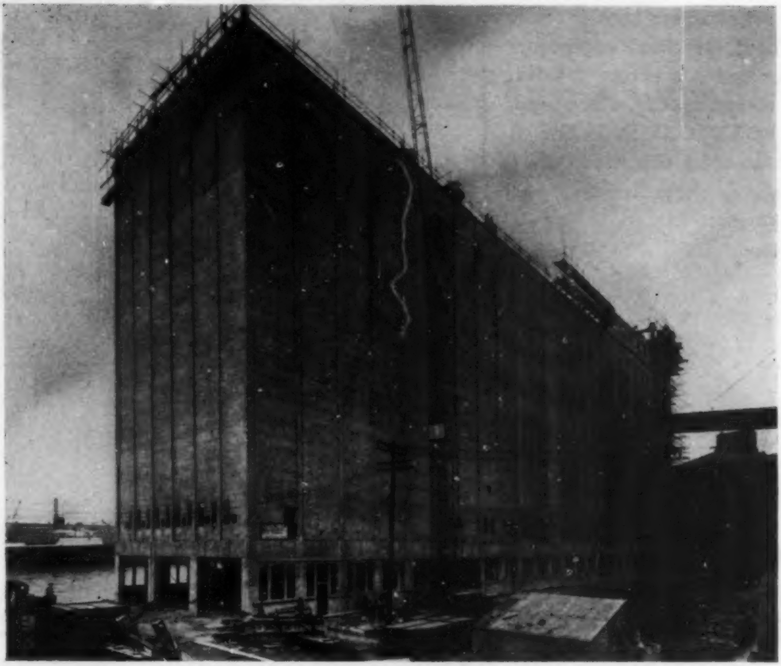

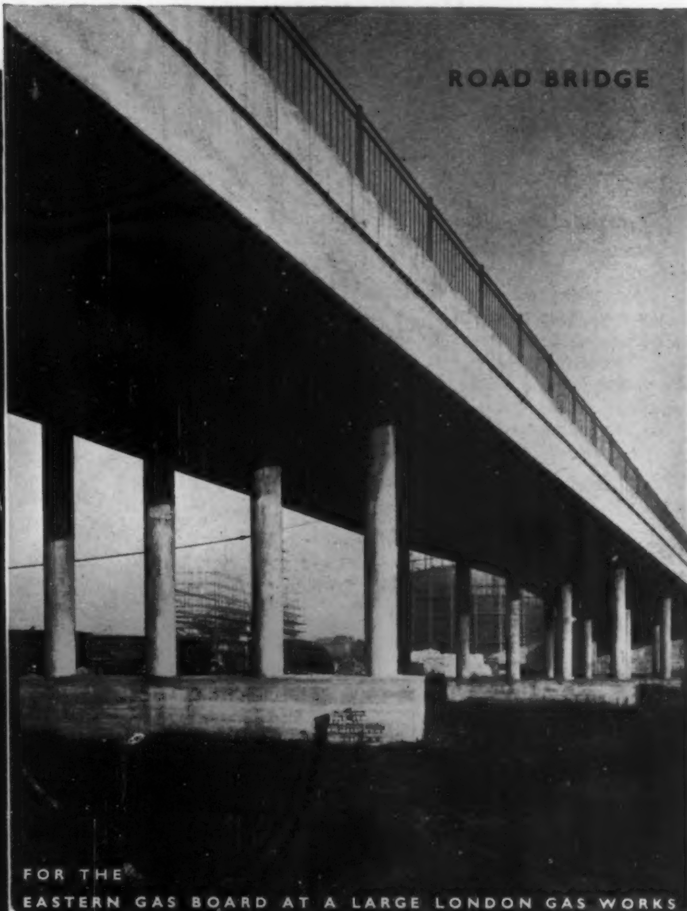
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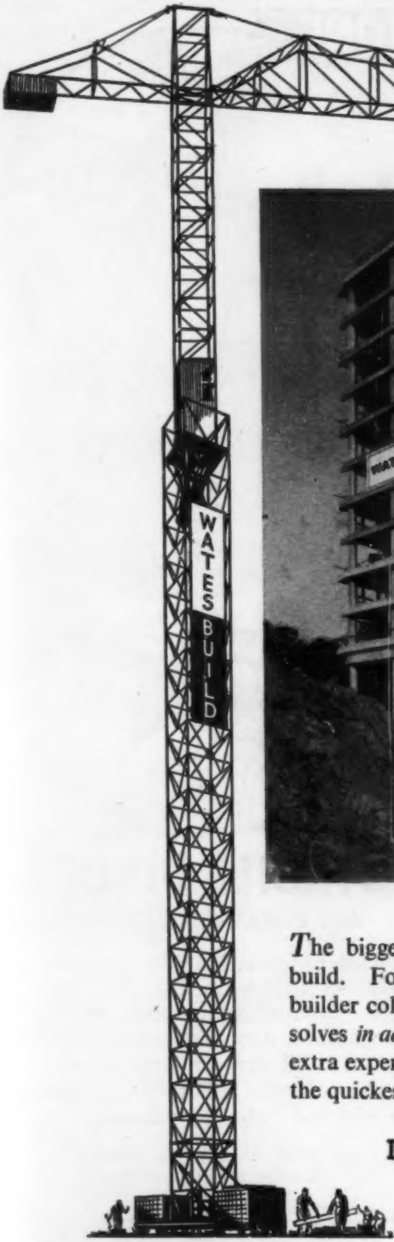
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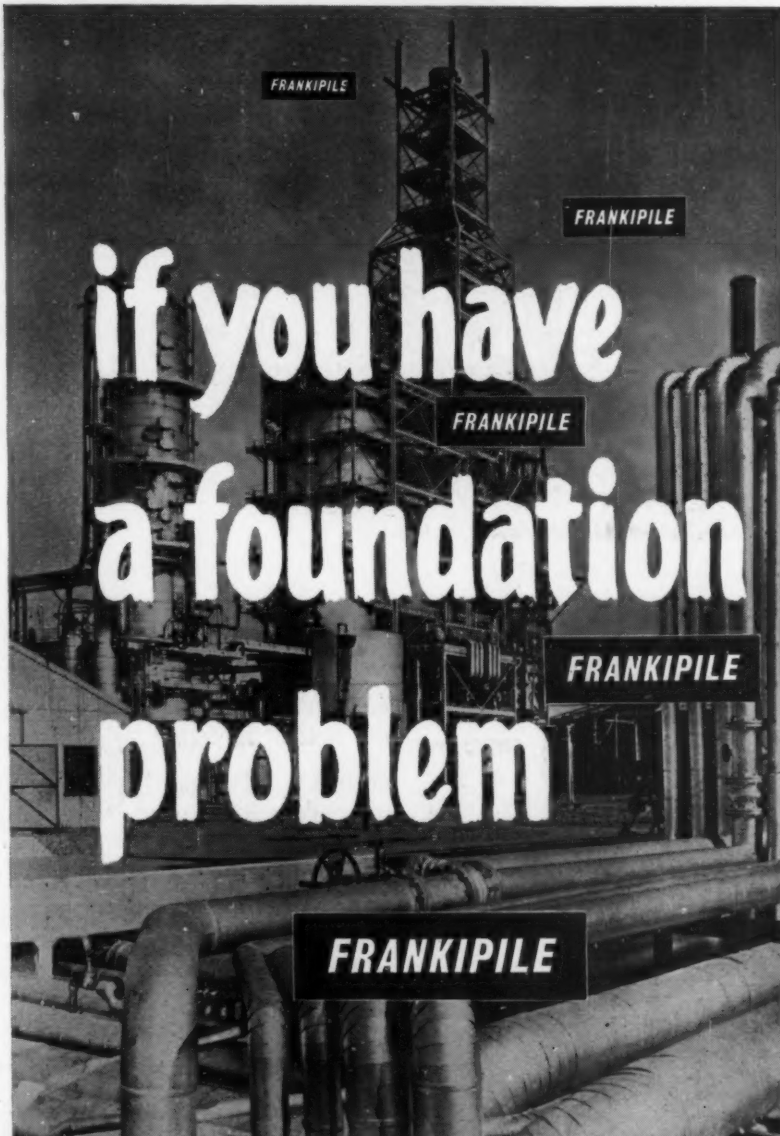

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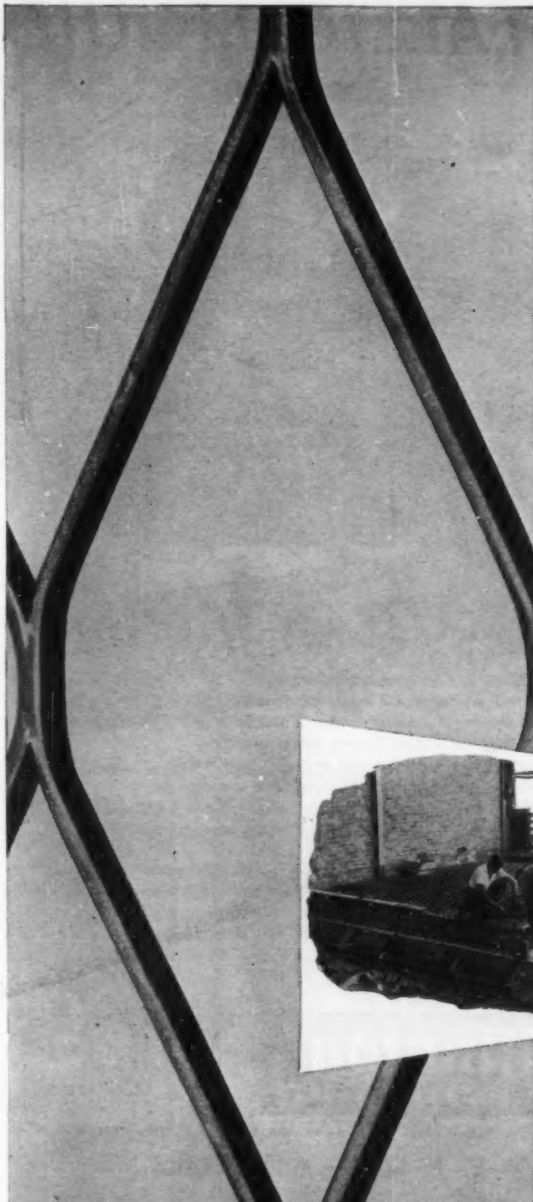

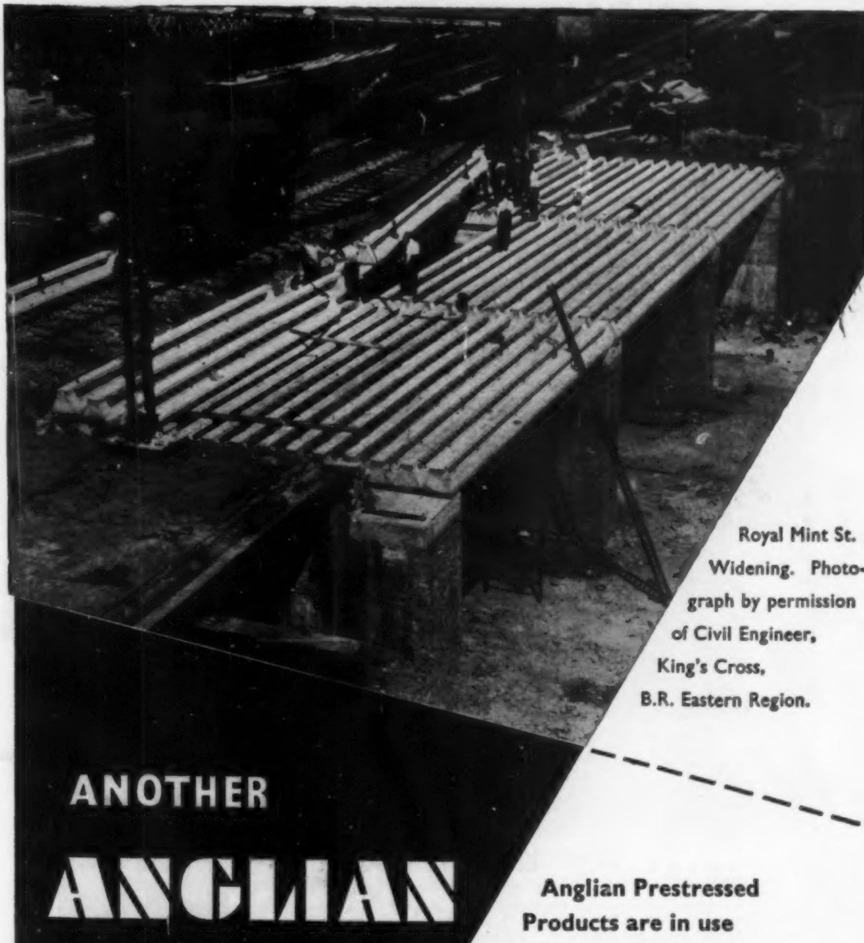

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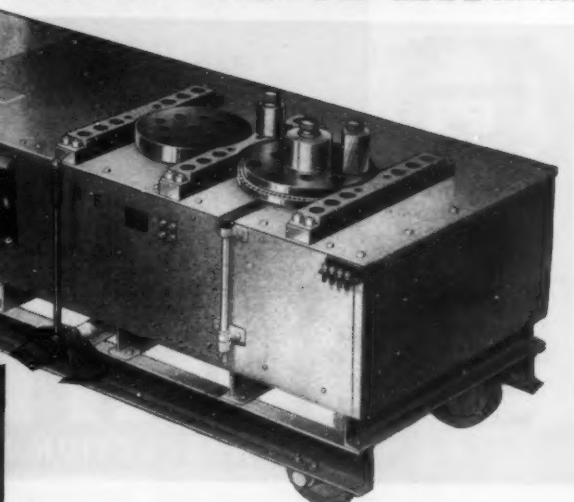

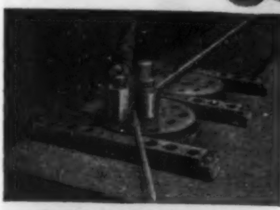

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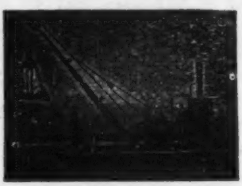

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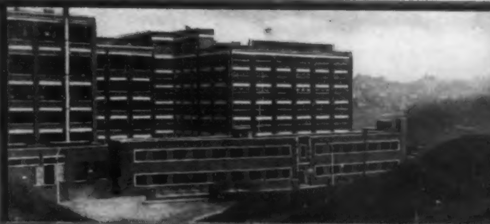

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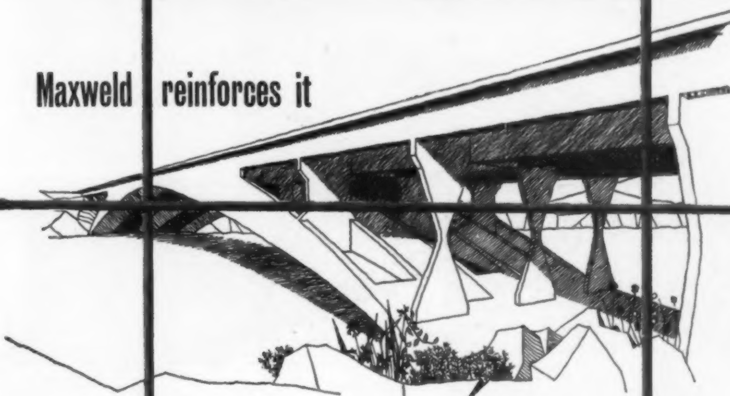

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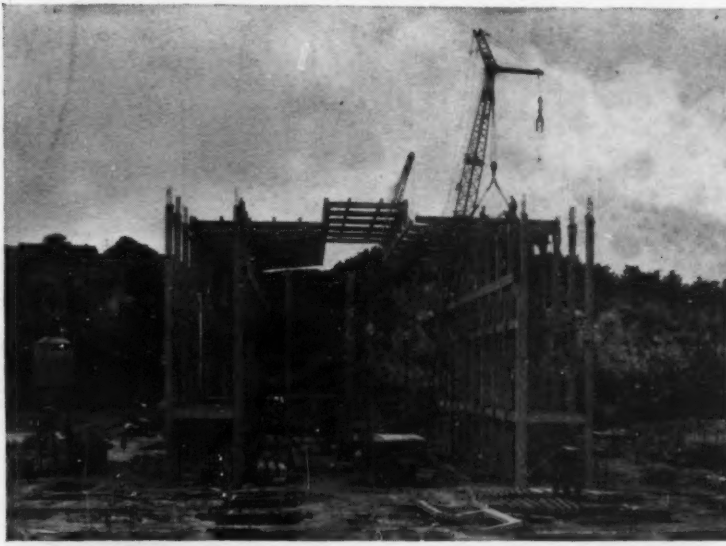

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Volume LII, No. 2. LONDON, FEBRUARY, 1957.

EDITORIAL NOTES

British Standard Meanings.

BECAUSE a good glossary of the terms used in concrete design and construction would be useful, it is a pity that a very poor one * has been issued and glorified with the title British Standard. It is stated that " terms whose meaning in general use is clear and which have no specialised meaning in relation to concrete are not as a rule included ", and it may be assumed that this is the reason for omitting some of the terms that are most often misused and consequently lead to confusion. It would, for example, have been useful if the meaning of reaction were defined as resistance for the information of those who talk and write about the reaction of a column on a footing and the reaction of the footing to the load imposed upon it, that is the use of the word reaction to describe both an active force and a passive resistance when in fact it should be used only in relation to the footing. A lecturer recently told us that he did so because his lecturer so misused the word when he was a student, and it had not occurred to him that it was senseless to use the same word with directly opposite meanings. Earth pressure could also be usefully defined in view of the frequency with which it is used to mean the resistance, not the pressure, of the earth beneath a foundation. We often read about a bending moment in a beam and a stress on a beam —these writers can have only a hazy idea of the meanings of these terms. If, with the arrogance of Humpty Dumpty, they claim that " When *I* use a word it means just what I choose it to mean ", then at least they should tell us that the more the reader knows of the English language the less he is likely to understand them.

One of the few such terms included is " punching shear ", which we thought had been laughed out of existence many years ago. It was used to describe the shearing stresses in a base due to a concentrated static load, generally of a column, and not to a dynamic load as is implied by the word punching. Many years ago we suggested that someone might care to make a sketch of a shear in fighting trim. Yet in this glossary the term is confidently put forward as one that may be used with the authority of the British Standards Institution, and is defined as " Shear stress calculated by dividing the load on a column by the product of its perimeter and the thickness of the base or cap ", so that a static column can apparently punch upwards as well as downwards and its

* British Standard No. 2787, " Glossary of Terms for Concrete and Reinforced Concrete ". Price 7s. 6d. net.

own weight is ignored. It would be useful to have a committee of people who know the meaning of words to consider new terms applied to new processes, so that such absurdities as pre-tensioned and post-tensioned concrete, bulk cement, and other recent innovations could be prevented from coming into common use.

A glossary is in the nature of a dictionary and in these days when little attention is given to English in the training of technicians it is desirable that a British Standard glossary should at least follow the rules of grammar. Yet we see " a hydraulic " used a dozen or so times, a reference to a " wood " tool, and the verb mix variously used as a noun and an adjective. A screed is defined as a layer of mortar, a straightedge for levelling a surface, the operation of forming a surface with the use of screeds, and the strips used as guides for the height of the straightedge. In fact the term screed means only the fixed guides for the straightedge, and it is very wrong to give it three other meanings to which it is not entitled and so add to the confusion that it should be the purpose of a glossary to remove.

A British Standard glossary should also be accurate, which this one is not. For example, alternate-bay construction is limited to slabs laid on the ground, whereas it is also correctly used to define the concreting of suspended slabs, walls, and any other work in which it is desired to reduce the effects of shrinkage. Under " Sliding Forms " we are referred to Climbing Forms, which sliding forms are not. Mass concrete is used when plain or unreinforced concrete is meant. A concrete vibrating machine is limited to one used to compact a ground slab, whereas such machines are used in all parts of concrete structures and for precast concrete products. Pulverised-fuel ash is mentioned only under its American description fly-ash, and is said to be recovered from the flues of power stations when in fact it is recovered from the precipitators before the gases enter the flue. Harsh concrete is defined as a mixture that is difficult to place due only to its grading or the texture of the aggregate, whereas harshness can also be due to the proportions of cement and water and the shape of the aggregate. Honeycombing is defined as interconnected large voids in concrete due to lack of mortar, whereas the voids need not be large, they need not be interconnected, they can result from causes other than lack of mortar, and they can appear on the surface as well as the interior. A pile cap is defined as a protective helmet on the top of a pile (which it is not) as well as a concrete block on top of a pile or a group of piles (which it is).

There are also some curiosities in this glossary. An ell-beam is defined, but not a tee-beam or any other shape of beam. Reference is made to British Standard Code of Practice No. 114 : 1956, whereas the latest available edition is dated 1948. There is an entry " Ferro-concrete—see Concrete ", and so we turn to " Concrete : Ferro-concrete—see Reinforced Concrete ", and then find " Reinforced concrete—see Concrete "—and nowhere is ferro-concrete defined.

It is stated in the foreword that the purpose of this glossary is to assist technical writers to use terms relating to concrete in their correct modern sense, and that this is one of a series that the Institution is preparing to cover the terms used in the building and associated industries. The document reviewed in the foregoing, and the looseness in the use of words and phraseology of some of the Standards issued by the British Standards Institution, do not indicate that this Institution is a suitable body to prepare glossaries.

Analysis of Bowstring Arches.

By F. H. TURNER, B.Sc.(Eng.), A.M.I.C.E., A.M.I.Struct.E., D.I.C.

A BOWSTRING arch is usually a highly indeterminate structure, and the analytical difficulties which it presents have hindered its more extensive use. A method of analysis which removes many of these difficulties is presented in the following.

An initial solution (termed the "particular solution") is first obtained, assuming that deflections of the tie-beam at the points of suspension are prevented. The particular solution is corrected by considering incompatibilities between the forces and extensions in the suspension rods. Tables are given for the case of a parabolic arch the second moment of area at any point of which is sec ψ times the second moment of area at the crown, where ψ is the angle of slope of the arch at the point in question. A numerical example is given.

Consider the arch shown in *Fig.* 1. The direct analysis would require the determination of at least ten redundancies, that is the solution of ten simultaneous linear equations. In the method described the number of equations is four.

Particular Solution.

The tie-beam is assumed to be continuous over non-yielding supports. It will be of assistance to imagine that this state is achieved by the method shown in *Fig.* 2.

STAGE I.—The loads on the arch are ignored at this stage, and the tie-beam

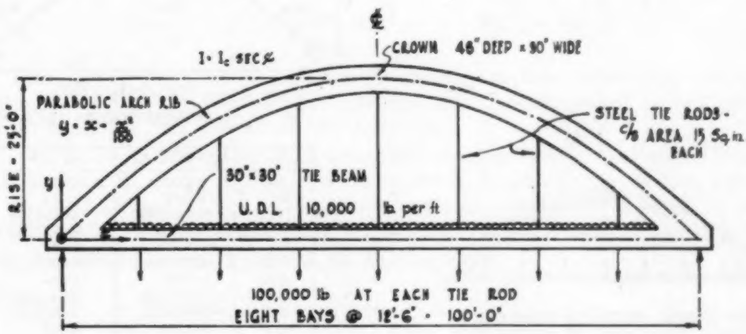


Fig. 1.

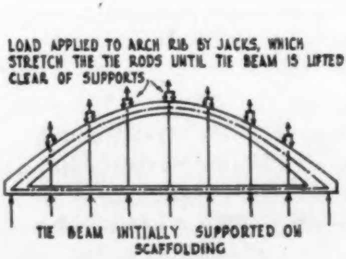


Fig. 2.

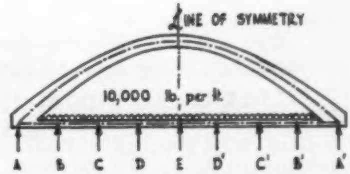
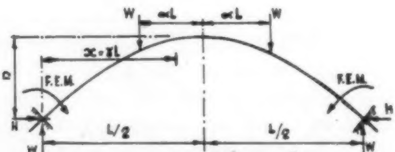


Fig. 3.

TABLE A.

A		B		C		D		E		Joint
AA'	AB	BA	BC	CB	CD	DC	DE	ED	ED'	Member
-½	½	½	½	½	½	½	½	½	½	C-O Factor
0·53	0·47	0·5	0·5	0·5	0·5	0·5	0·5	0·5	0·5	Dist^n Factor
0	-1563	+1563	-1563	+1563	-1563	+1563	-1563	+1563	-1563	F. E. Moment
+829	+734	·	·	·	·	·	·	·	·	balance
+276	·	+367	·	·	·	·	·	·	·	Carry-Over
-146	-130	-184	-183	·	·	·	·	·	·	Balance
-49	-92	-65	·	-92	·	·	·	·	·	Carry-Over
+75	+66	+38	+32	+46	+46	·	·	·	·	Balance
+25	+16	+33	+23	+16	·	+23	·	·	·	Carry-Over
-22	-19	-28	-28	-8	-8	-12	-11	·	·	Balance
-7	-14	-10	-4	-14	-6	-4	·	-6	+6	Carry-Over
+11	+10	+7	+7	+10	+10	+2	+2	·	·	Balance
+4	+4	+5	+5	+3	+1	+5	·	+1	-1	Carry-Over
-4	-4	-5	-5	-2	-2	-3	-2	·	·	Balance
+992	-992	+1716	-1716	+1522	-1522	+1574	-1574	+1558	-1558	Total Moment
·	62,500	62,500	62,500	62,500	62,500	62,500	62,500	62,500	62,500	Free Reaction (lb)
·	-4,805	+4,805	+1,275	-1,275	-345	+345	+105	-105	-105	Continuity R^n (lb)
57,695		231,080		223,380		225,450		224,790		Total Reaction (lb)

(Moments in 1000 in.-lb. units.)

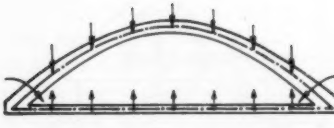


$$F.E.M. = WL\left(\tfrac{1}{16} - \tfrac{3\alpha^2}{2} + 5\alpha^4\right)$$

$$H = \tfrac{3}{2h}\left(\text{SUPPORT MOMENT} + WL\left[\tfrac{1}{4} - \alpha^2\right]\right)$$

α	$\frac{1}{16} - \frac{3\alpha^2}{2} + 5\alpha^4$	$\frac{1}{4} - \alpha^2$
0	0·0625	0·25
0·1	0·048	0·24
0·125	0·040285	0·234375
0·167	0·024691	0·222222
0·2	0·0105	0·21
0·25	-0·011719	0·1875
0·3	-0·024	0·16
0·333	-0·042438	0·138888
0·375	-0·049561	0·109375
0·4	-0·0495	0·09
0·5	0	0

Fig. 5.



ARCH LOADING PRODUCES FIXED END MOMENTS, WHICH MUST BE DISTRIBUTED THROUGH TIE BEAM.

Fig. 4.

I

When $\alpha = 0$, $(1/16 - 3\alpha^2/2 + 5\alpha^4) = 0{\cdot}0625$; When $\alpha = 0{\cdot}125$, $(1/16 - 3\alpha^2/2 + 5\alpha^4) = 0{\cdot}0403$;

When $\alpha = 0{\cdot}250$, $(1/16 - 3\alpha^2/2 + 5\alpha^4) = -0{\cdot}0117$; When $\alpha = 0{\cdot}375$, $(1/16 - 3\alpha^2/2 + 5\alpha^4) = -0{\cdot}0496$

$$\therefore F.E.M.\ (\text{at L.H.E.}) = 1200\left\{(1/2 \times 224{,}790 \times 0{\cdot}0625) + (225{,}450 \times 0{\cdot}0403) - (223{,}380 \times 0{\cdot}0117) - (231{,}080 \times 0{\cdot}0496)\right\} = 2{,}440{,}000 \text{ lb.in}$$

is considered as the continuous beam shown in *Fig.* 3. This is conveniently analysed by moment distribution. In the case of a parabolic arch, when $I = I_c \sec \psi$, the carry-over factor is one-third and the stiffness is $\frac{9EI_c}{L}$, in which I_c is the second moment of area at the crown of the arch (see this journal for April 1956). Therefore

$$I_c = \frac{30 \times 48^3}{12} = 276{,}500 \text{ in.}^4 \quad L_{\text{arch}} = 1200 \text{ in.} \quad \frac{I_c}{L} = 230.$$

$$I_{\text{beam}} = \frac{30 \times 30^3}{12} = 67{,}500 \text{ in.}^4 \quad L_{\text{beam}} = 150 \text{ in.} \quad \frac{I}{L} = 450.$$

The stiffness of the beam $= \frac{4EI}{L}$.

The distribution factors are therefore:

To AA′ and A′A, $\frac{9 \times 230}{(9 \times 230) + (4 \times 450)} = 0{\cdot}53.$

To AB and A′B′, $\frac{4 \times 450}{(9 \times 230) + (4 \times 450)} = 0{\cdot}47.$

To all other spans, 0·5.

The fixed-end moments of the beam are

$$\pm \frac{wl^2}{12} = \pm 10{,}000 \times 12{\cdot}5^2 = \pm 1{,}563{,}000 \text{ in.-lb.}$$

Referring to *Figs.* 2 and 3, the moments and reactions (*Table* A) correspond to the case where the tie-beam is fully loaded and is supported on scaffolding, and no loads are yet placed on the arch rib.

STAGE II.—Consider the loads in the suspension rods to be applied to the arch. This produces fixed-end moments in the arch which must be distributed through the tie-beam (*Fig.* 4). The fixed-end moments at each end of a parabolic arch, when $I = I_c \sec \psi$, due to two equal concentrated loads W spaced symmetrically about the centre-line and distant αL from it, are

$$\text{F.E.M.}_{(\text{L.H.E.})} = WL\left(\frac{1}{16} - \frac{3\alpha^2}{2} + 5\alpha^4\right)$$

(assuming clockwise moments to be positive).

This and all succeeding formulæ apply only to the case of a parabolic arch when $I = I_c \sec \psi$. A central load is assumed to be two coincident loads (W). Values of $\left(\frac{1}{16} - \frac{3\alpha^2}{2} + 5\alpha^4\right)$ are tabulated in *Fig.* 5, and the fixed-end moment is obtained in equation 1. This moment must be balanced and distributed throughout the frame as shown in *Table* B.

It is necessary to appreciate the physical meaning of these figures with reference to *Fig.* 2. The effect of applying to the arch the loads ascertained in Stage I is to deform both the arch and the tie-beam in such a way that the bending moments and forces in the frame are altered by the amounts shown in *Table* B.

TABLE B.

A		B		C		D		E		Joint
AA'	AB	BA	BC	CB	CD	DC	DE	ED	ED'	Member
$-1/3$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	C-O Factor
0·53	0·47	0·5	0·5	0·5	0·5	0·5	0·5	0·5	0·5	Distn Factor
+2440	·	·	·	·	·	·	·	·	·	F. E. Moment
-1294	-1146	·	·	·	·	·	·	·	·	Balance
-431	·	-573	·	·	·	·	·	·	·	Carry-Over
+229	+202	+287	+286	·	·	·	·	·	·	Balance
+76	+143	+101	·	+143	·	·	·	·	·	Carry-Over
-116	-103	-51	-50	-72	-71	·	·	·	·	Balance
-39	-26	-52	-36	-25	·	-36	·	·	·	Carry-Over
+35	+30	+44	+44	+13	+12	+18	+18	·	·	Balance
+12	+22	+15	+6	+22	+9	+6	·	+9	-9	Carry-Over
-18	-16	-10	-11	-15	-16	-3	-3	·	·	Balance
-6	-5	-8	-8	-6	-2	-8	·	-2	+2	Carry-Over
+6	+5	+8	+8	+4	+4	+4	+4	·	·	Balance
+894	-894	-239	+239	+64	-64	-19	+19	+7	-7	Total Moment
·	+7550	-7550	-2020	+2020	+555	-555	-175	+175	+175	Reaction Change
+7,550		-9,550		+2575		-730		+350		Total Change (N)

(Moments in 1000 in.-lb. units.)

In particular, the modifications to the loads on the tie-rods will alter the fixed-end moments for the arch as follows.

Alteration in fixed-end moment

$$= 1200\left(\frac{350}{2} \times 0{\cdot}0625 - 730 \times 0{\cdot}0403 - 2575 \times 0{\cdot}0117 + 9550 \times 0{\cdot}0496\right)$$

$$= 509{,}000 \text{ in.-lb.}$$

In other words, the effect of applying to the arch and distributing a fixed-end moment of $+ 2{,}440{,}000$ in.-lb. is to produce another fixed-end moment of $+509{,}000$ in.-lb. Clearly this in turn will produce a further fixed-end moment of $\frac{509}{2440} \times 509{,}000$ in.-lb., and so on. The total fixed-end moment to be considered is therefore the sum to infinity of the series

$$2{,}440{,}000\ (1 + r + r^2 + r^3 + \ldots), \text{ in which } r = \frac{509}{2440} = 0{\cdot}209.$$

The expression in brackets is a geometrical progression whose sum is $S_\infty = \frac{1}{1 - r}$, that is $S_\infty = \frac{1}{1 - 0{\cdot}209} = \frac{1}{0{\cdot}791} = 1{\cdot}265$. The total fixed-end moment is therefore $2{,}440{,}000 \times 1{\cdot}265 = 3{,}085{,}000$ in.-lb., and the bending moment and tie-load corrections must be increased in the ratio $1{\cdot}265 : 1$.

The particular solution is therefore as shown in *Table* C. In many cases this solution may be sufficiently accurate, as it over-estimates the load on the arch. It should also be noted that the assumed method of construction is feasible in cases where the dead load is appreciably greater than the live load. However, if the lengths of the tie-rods cannot be altered, or if the live load is greater than the dead load, or the size of the structure renders a more accurate analysis desirable, the particular solution should be corrected to allow for the fact that the shortening of the tie-rods implicit in *Fig.* 2 cannot occur.

TABLE C.

BENDING MOMENTS

A		B		C		D		E		Joint
AA'	AB	BA	BC	CB	CD	DC	DE	ED	ED'	Member
+992	-992	+1,716	-1,716	+1,522	-1,522	+1,574	-1,574	+1,558	-1,558	Stage 1
+1,130	-1,130	- 302	+ 302	+ 81	- 81	- 24	+ 24	+ 9	- 9	Stage 2 = 1·265
+2,122	-2,122	+1,411	-1,411	+1,603	-1,603	+1,550	-1,550	+1,567	-1,567	Total Moment

LOADS IN TIE-RODS

B	C	D	E	Tie-rod
231,080	223,380	225,450	224,790	Stage 1
- 12,080	+ 3,260	- 920	+ 440	Stage 2 = 1·265
219,000	226,640	224,530	225,230	Total Load (lb)

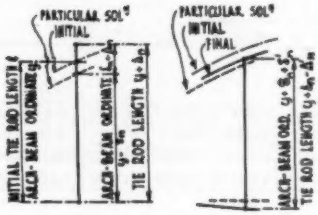


Fig. 6.

The horizontal tension in the tie-beam may be obtained from

$$H = \frac{3}{2h}\left[\text{restraint moment} + L\sum W(\tfrac{1}{4}-\alpha^2)\right]$$

$$= \frac{3}{2\times 25\times 12}\left[2{,}122{,}000 + 1200\left(\frac{225{,}230}{2}\times\tfrac{1}{4} + 224{,}530\times\frac{15}{64}\right.\right.$$

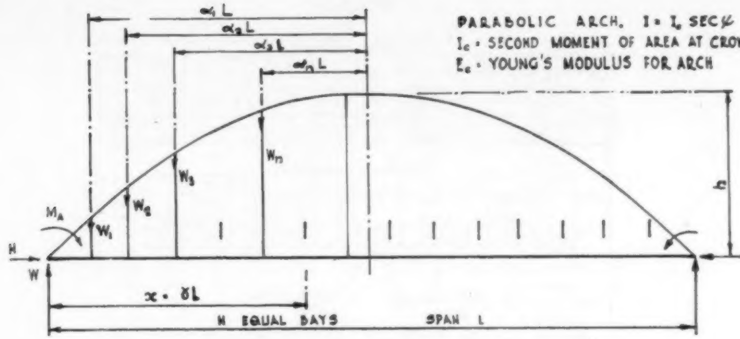
$$\left.\left. + 226{,}640\times\frac{3}{16} + 219{,}000\times\frac{7}{64}\right)\right]$$

$= 887{,}960$ lb. [See *Fig.* 5 for values of $(\frac{1}{4}-\alpha^2)$.]

The vertical load from the arch is $112{,}615 + 224{,}530 + 226{,}640 + 219{,}000 = 782{,}785$ lb.

Correction of the Particular Solution.

Refer to *Figs.* 2 and 6. The initial (unstrained) length of any tie-rod is assumed to be the same as the distance between the centre-lines of the tie-beam and the arch, and is $y = 4h\left(\frac{x}{l}-\frac{x^2}{l^2}\right)$. When the arch is loaded, as assumed in the foregoing, the distance between the arch and the beam decreases by an amount δ_n while the length of the tie-rod increases by an amount Δ_n, and therefore the distance between the end of the tie-rod and the arch is $\Delta_n + \delta_n$. If this relative movement between the arch and tie-rod is not possible it may be imagined that the scaffold beneath the tie-beam is removed and that the load applied by the jacks is reduced until the end of the tie-rod again coincides with the centre-line of the arch. If it is assumed that the load is reduced by an amount R_n, that



γ	0	0·1	0·125	0·1667	0·2	0·25	0·3	0·3333	0·375	0·4	0·5
$\gamma^2-\gamma$	0	-0·09	-0·109375	-0·138888	-0·16	-0·1875	-0·21	-0·222222	-0·234375	-0·24	-0·25
$\gamma^3/6-\gamma^4/12-\gamma/12$	0	-0·008175	-0·0101115	-0·013182	-0·015467	-0·018555	-0·021175	-0·022634	-0·024109	-0·0248	-0·026042
$\gamma^3/3-\gamma/4$	0	-0·024667	-0·030599	-0·0401235	-0·047333	-0·057292	-0·066	-0·070988	-0·076172	-0·078667	-0·083333

VALUES OF $\alpha_n^2.\gamma$ — VALUES OF $[\gamma-(1/2-\alpha_n)]^3$

N = 2

α_n	γ=0·5
0	0

α_n	γ=0·5
0	0

N = 3

α_n	γ=0·333
0·167	0·00925

α_n	γ=0·333
0·167	0

N = 4

α_n γ=	0·25	0·5
0	0	0
0·25	0·015625	0·03125

α_n γ=	0·25	0·5
0	0	0
0·25	0	0·015625

N = 5

α_n γ=	0·2	0·4
0·1	0·002	0·004
0·3	0·018	0·036

α_n γ=	0·2	0·4
0·1	0	0
0·3	0	0·008

N = 6

α_n γ=	0·167	0·333	0·5
0	0	0	0
0·167	0·004629	0·00925	0·0138
0·333	0·0185	0·037	0·05

α_n γ=	0·167	0·333	0·5
0	0	0	0
0·167	0	0	0·004629
0·333	0	0·004629	0·037

N = 8

α_n γ=	0·125	0·25	0·375	0·5
0	0	0	0	0
0·125	0·001953	0·003906	0·005859	0·007813
0·25	0·007813	0·015625	0·023438	0·03125
0·375	0·017578	0·035156	0·052734	0·070313

α_n γ=	0·125	0·25	0·375	0·5
0	0	0	0	0
0·125	0	0	0	0·001953
0·25	0	0	0·001953	0·015625
0·375	0	0·001953	0·015625	0·052734

N = 10

α_n γ=	0·1	0·2	0·3	0·4	0·5
0	0	0	0	0	0
0·1	0·001	0·002	0·003	0·004	0·005
0·2	0·004	0·008	0·012	0·016	0·02
0·3	0·009	0·018	0·027	0·036	0·045
0·4	0·016	0·032	0·048	0·064	0·08

α_n γ=	0·1	0·2	0·3	0·4	0·5
0	0	0	0	0	0
0·1	0	0	0	0	0·001
0·2	0	0	0	0·001	0·008
0·3	0	0	0·001	0·008	0·027
0·4	0	0·001	0·008	0·027	0·064

Fig. 7.

$$\frac{M_A}{2L} = \frac{2{,}122{,}000}{2{,}400} = 884 \text{ lb} \qquad \frac{4.H_A.h}{L} = \frac{4 \times 25 \times 887{,}960}{100} = 887{,}960 \text{ lb}$$

II

$\gamma = 0{\cdot}125$:

$$E_c.I_c.\delta_1/L^3 = -(884 \times 0{\cdot}109375) + (887{,}960 \times 0{\cdot}010111) - \left(\frac{782{,}785}{2} \times 0{\cdot}030599\right) + \left\{\left(\frac{219{,}000}{2} \times 0{\cdot}017578\right) + \left(\frac{226{,}640}{2} \times 0{\cdot}007813\right) + \left(\frac{224{,}530}{2} \times 0{\cdot}001953\right)\right\} - (0)$$

$$= -96{\cdot}6 + 8{,}978{\cdot}2 - 11{,}976{\cdot}2 + 3{,}030{\cdot}2 = \underline{-64{\cdot}4}$$

$\gamma = 0{\cdot}25$:

$$E_c.I_c.\delta_2/L^3 = -(884 \times 0{\cdot}1875) + (887{,}960 \times 0{\cdot}018555) - \left(\frac{782{,}785}{2} \times 0{\cdot}057292\right) + (2 \times 3{,}030{\cdot}2) - \left(\frac{219{,}000}{6} \times 0{\cdot}001953\right) = \underline{-124{\cdot}3}$$

$\gamma = 0{\cdot}375$:

$$E_c.I_c.\delta_3/L^3 = -(884 \times 0{\cdot}234375) + (887{,}960 \times 0{\cdot}024109) - \left(\frac{782{,}785}{2} \times 0{\cdot}076172\right) + (3 \times 3{,}030{\cdot}2) - \left\{\left(\frac{219{,}000}{6} \times 0{\cdot}015625\right) + \left(\frac{226{,}640}{6} \times 0{\cdot}001953\right)\right\} = \underline{-166{\cdot}0}$$

$\gamma = 0{\cdot}5$:

$$E_c.I_c.\delta_4/L^3 = -(884 \times 0{\cdot}25) + (887{,}960 \times 0{\cdot}026042) - \left(\frac{782{,}785}{2} \times 0{\cdot}083333\right) + (4 \times 3{,}030{\cdot}2) - \left\{\left(\frac{219{,}000}{6} \times 0{\cdot}052734\right) + \left(\frac{226{,}640}{6} \times 0{\cdot}015625\right) + \left(\frac{224{,}530}{6} \times 0{\cdot}001953\right)\right\} = \underline{-189{\cdot}9}$$

the extension of the tie-rod is reduced by Δ'_n, and that the distance between the centre-lines of the arch and beam increases from $y - \delta_n$ to $(y - \delta_n) + \delta'_n$, then $(\Delta_n + \delta_n) = (\Delta'_n + \delta'_n)$, or $\delta'_n = \delta_n + \Delta_n - \Delta'_n$.

Hence it is necessary to determine the forces that would increase the distance between the arch and the beam by an amount $\delta_n + \Delta_n - \Delta'_n$ at each tie-rod, and the superposition of these values on those of the particular solution will give the final solution. It is not difficult to show that

$$\frac{E_c I_c \delta_n}{L^3} = \frac{M_A}{2L}(\gamma^2 - \gamma) - \frac{4H_A h}{L}\left(\frac{\gamma^3}{6} - \frac{\gamma^4}{12} - \frac{\gamma}{12}\right) + \frac{W}{2}\left(\frac{\gamma^3}{3} - \frac{\gamma}{4}\right) + \sum_{n=0}^{n=\frac{N}{2}} \frac{W_n}{2}.\alpha_n^2\gamma - \sum_{n=0}^{n=N\gamma} \frac{W_n}{6}[\gamma - (\tfrac{1}{2} - \alpha_n)]^3,$$

where the symbols are as defined in *Fig.* 7.

Tables are also given in *Fig.* 7 for different values of γ and α, which reduce labour in evaluating the expression. The calculation of $\frac{E_c I_c \delta_n}{L^3}$ is shown in equations II; it involves a small difference between two large numbers, and should therefore be made carefully.

The extension of each tie-rod is given by $\Delta_n = \frac{W_n L_r}{A_r E_r}$, in which L_r, A_r, and E_r are length, cross-sectional area, and modulus of elasticity of the rod.

Consider *Fig.* 8. The fixed-end moments for the arch are given by

$$\text{F.E.M.}_{(\text{L.H.E.})} = -RL\left(\frac{1}{16} - \frac{3\alpha^2}{2} + 5\alpha^4\right) \text{ (clockwise moments are positive).}$$

For the beam, $\text{F.E.M.}_{(\text{L.H.E.})} = -RL(\frac{1}{4} - \alpha^2)$.

Assuming distribution factors of k (for the arch) and $1 - k$ (for the beam), the moments at joints A and A′ can be distributed as shown in *Table* D. Using this value for M'_A,

$$H' = \frac{3}{2h}.RL(\tfrac{1}{4} - \alpha^2)\left(\frac{15(k-1)(\frac{1}{4} - \alpha^2)}{5k+3}\right).$$

TABLE D.

A		Joint
AA′ (Arch)	AA′ (Beam)	Member
k	$1-k$	Distⁿ Factor
$-1/3$	$1/2$	C-O Factor
$-RL(1/16 - 3\alpha^2/2 + 5\alpha^4)$	$RL(\alpha^2 - 1/4)$	F. E. Moment
Total: $RL(-5/16 + 5\alpha^2/2 - 5\alpha^4)$		
$-kRL(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	$-(1-k)RL(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	Balance
$-k/3.RL(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	$+1/2(1-k)RL(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	C-O
$-kRL(1/2 - 5k/6)(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	$-(1-k)RL(1/2 - 5k/6)(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	Balance
$-k/3.RL(1/2 - 5k/6)(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	$+1/2(1-k)RL(1/2 - 5k/6)(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	C-O
$-kRL(1/2 - 5k/6)^2(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	$-(1-k)RL(1/2 - 5k/6)^2(-5/16 + 5\alpha^2/2 - 5\alpha^4)$	Balance
etc.	etc.	etc.
These terms constitute infinite geometric series, the sums of which are:		
$RL(1/4 - \alpha^2)\left(1 + \frac{15[k-1][1/4 - \alpha^2]}{5k+3}\right)$	$-RL(1/4 - \alpha^2)\left(1 + \frac{15[k-1][1/4 - \alpha^2]}{5k+3}\right)$	Total M'_A

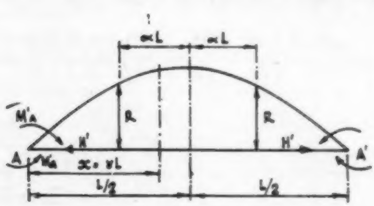


Fig. 8.

III

$$I_{arch} = 276{,}500 \text{ in}^4; \quad I_{beam} = 67{,}500 \text{ in}^4$$

$$\therefore k = \frac{9 \times 276{,}500}{(9 \times 276{,}500) + (4 \times 67{,}500)} = 0{\cdot}903; \quad 1-k = 0{\cdot}097$$

$$\therefore M'_A = RL(1/4 - \alpha^2)\left\{1 + \frac{15 \times -0{\cdot}097 \times (1/4 - \alpha^2)}{4{\cdot}515 + 3}\right\} = RL(1/4 - \alpha^2)(1 - 0{\cdot}194[1/4 - \alpha^2])$$

$$\text{and } H' = \frac{3 \times 100}{2 \times 25} \cdot R(1/4 - \alpha^2)(-0{\cdot}194[1/4 - \alpha^2]) = 6R(1/4 - \alpha^2)(-0{\cdot}194[1/4 - \alpha^2])$$

IV

$$\frac{E_c.I_c.I_b.\delta'_n}{(I_b+I_c).L^3} = \sum_{n=0}^{n=N/2}\frac{M'_A}{2L}(\gamma^2-\gamma) + \sum_{n=0}^{n=N/2}\frac{4hH'}{L}.\frac{I_b}{I_b+I_c}.\left(\frac{\gamma^3}{6}-\frac{\gamma^4}{12}-\frac{\gamma}{12}\right) - \sum_{n=0}^{n=N/2}\frac{R}{2}\left(\frac{\gamma^2}{3}-\frac{\gamma}{4}\right)$$

$$+ \sum_{n=0}^{n=N/3}\frac{R_n}{6}\left[\gamma-\left(\frac{1}{2}-\alpha_n\right)\right]^3 - \sum_{n=0}^{n=N/2}\frac{R_n}{2}.\alpha_n^2.\gamma$$

Now $$\sum_{n=0}^{n=N/2}\frac{M'_A}{2L} = \sum_{n=0}^{n=N/2}\frac{R_n}{2}\left(\frac{1}{4}-\alpha_n^2\right)\left(1-0{\cdot}194\left[\frac{1}{4}-\alpha_n^2\right]\right)$$

$$= 0{\cdot}5\{[R_1\times 0{\cdot}109375\times(1-[0{\cdot}194\times 0{\cdot}109375])] + [R_2\times 0{\cdot}1875\times(1-[0{\cdot}194\times 0{\cdot}1875])]$$
$$+[R_3\times 0{\cdot}234375\times(1-[0{\cdot}194\times 0{\cdot}234375])] + [0{\cdot}5\times R_4\times 0{\cdot}25\times(1-[0{\cdot}194\times 0{\cdot}25])]\}$$

$$= 0{\cdot}053527R_1 + 0{\cdot}090340R_2 + 0{\cdot}111859R_3 + 0{\cdot}059469R_4$$

and $$\sum_{n=0}^{n=N/2}\frac{4h.\ I_b\ \ .H'}{L\ (I_b+I_c)} = \frac{67{,}500}{276{,}500+67{,}500}\sum_{n=0}^{n=N/2}6R_n\left(\frac{1}{4}-\alpha_n^2\right)\left(0{\cdot}194\left[\frac{1}{4}-\alpha_n^2\right]\right)$$

$$= \frac{6\times 67{,}500\times 0{\cdot}194}{344{,}000}\left(0{\cdot}109375^2R_1 + 0{\cdot}1875^2R_2 + 0{\cdot}234375R_3 + \frac{0{\cdot}25^2}{2}R_4\right)$$

$$= 0{\cdot}002732R_1 + 0{\cdot}008030R_2 + 0{\cdot}012546R_3 + 0{\cdot}007138R_4$$

$$\therefore \frac{E_c.I_c.I_b.\delta'_n}{(I_b+I_c).L^3} = (0{\cdot}053527R_1 + 0{\cdot}090340R_2 + 0{\cdot}111859R_3 + 0{\cdot}059469R_4)(\gamma^2-\gamma)$$
$$+ (0{\cdot}002732R_1 + 0{\cdot}008030R_2 + 0{\cdot}012546R_3 + 0{\cdot}007138R_4)(\gamma^3/6-\gamma^4/12-\gamma/12)$$
$$- \left(\frac{R_1}{2}+\frac{R_2}{2}+\frac{R_3}{2}+\frac{R_4}{4}\right)\left(\frac{\gamma^3}{3}-\frac{\gamma}{4}\right) + \sum_{n=0}^{n=N/2}\frac{R_n}{6}\left[\gamma-\left(\frac{1}{2}-\alpha_n\right)\right]^3 - \sum_{n=0}^{n=N/2}\frac{R_n}{2}\alpha_n^2.\gamma$$

TABLE E.

n	L_r	$\frac{L_r}{A_r.E_r}$	Δ'_n	W_n	Δ_n
1	131·25"	$(7/6E_c)$	$7R_1/6E_c$	219,000	$255{,}500/E_c$
2	225·0"	$(2/E_c)$	$2R_2/E_c$	226,640	$453{,}280/E_c$
3	281·25"	$(2{\cdot}5/E_c)$	$2{\cdot}5R_3/E_c$	224,530	$561{,}325/E_c$
4	300·0"	$(8/3E_c)$	$8R_4/3E_c$	225,230	$600{,}640/E_c$

TABLE F.

Loads on tie-rods (in.-lb.)

B	C	D	E	Joint
219,000	226,640	224,530	225,230	Particular Sn
− 35,685	+ 1,517	+ 4,360	− 7,132	Correction
183,315	228,157	228,890	218,098	Total (lb)

TABLE G.

Bending moments in tie-beam in 1000 in.-lb. units.

A		B		C		D		E		Joint
AA'	AB	BA	BC	CB	CD	DC	DE	ED	ED'	Member
+2122	−2122	+1411	−1411	+1603	−1603	+1550	−1550	+1567	−1567	Particular Soln
+4103	−4103	−903	+903	−556	+556	−437	+437	−972	+972	Correction
+6225	−6225	+508	−508	+1047	−1047	+1113	−1113	+595	−595	Total Moment

When $\gamma = 0{\cdot}125$,

$$\frac{E_c . I_c . I_b . \delta_1'}{(I_c + I_b) L^3} = -(0{\cdot}053527 R_1 + 0{\cdot}090340 R_2 + 0{\cdot}111859 R_3 + 0{\cdot}059469 R_4) \times 0{\cdot}109375$$
$$-(0{\cdot}002732 R_1 + 0{\cdot}008030 R_2 + 0{\cdot}012546 R_3 + 0{\cdot}007138 R_4) \times 0{\cdot}010111$$
$$+[0{\cdot}5 \times (R_1 + R_2 + R_3 + 0{\cdot}5 R_4)] \times 0{\cdot}030599 - [0{\cdot}5 \times (0{\cdot}017578 R_1 + 0{\cdot}007813 R_2 + 0{\cdot}001953 R_3)]$$
$$= 0{\cdot}000628 R_1 + 0{\cdot}001431 R_2 + 0{\cdot}001962 R_3 + 0{\cdot}001073 R_4$$

When $\gamma = 0{\cdot}25$,

$$\frac{E_c . I_c . I_b . \delta_2'}{(I_c + I_b) L^3} = -(0{\cdot}053527 R_1 + 0{\cdot}090340 R_2 + 0{\cdot}111859 R_3 + 0{\cdot}059469 R_4) \times 0{\cdot}1875$$
$$-(0{\cdot}002732 R_1 + 0{\cdot}008030 R_2 + 0{\cdot}012546 R_3 + 0{\cdot}007138 R_4) \times 0{\cdot}018555$$
$$+[0{\cdot}5 \times (R_1 + R_2 + R_3 + 0{\cdot}5 R_4)] \times 0{\cdot}057292 - [0{\cdot}5 \times (0{\cdot}035156 R_1 + 0{\cdot}015625 R_2 + 0{\cdot}003906 R_3)] + [\tfrac{R_1}{6} \times 0{\cdot}001953]$$
$$= 0{\cdot}001307 R_1 + 0{\cdot}003746 R_2 \mp 0{\cdot}005487 R_3 + 0{\cdot}003040 R_4$$

V

When $\gamma = 0{\cdot}375$,

$$\frac{E_c . I_c . I_b . \delta_3'}{(I_c + I_b) L^3} = -(0{\cdot}053527 R_1 + 0{\cdot}090340 R_2 + 0{\cdot}111859 R_3 + 0{\cdot}059469 R_4) \times 0{\cdot}234375$$
$$-(0{\cdot}002732 R_1 + 0{\cdot}008030 R_2 + 0{\cdot}012546 R_3 + 0{\cdot}007138 R_4) \times 0{\cdot}024109$$
$$+[0{\cdot}5 \times (R_1 + R_2 + R_3 + 0{\cdot}5 R_4)] \times 0{\cdot}076172 - [0{\cdot}5 \times (0{\cdot}052734 R_1 + 0{\cdot}023438 R_2 + 0{\cdot}005859 R_3)] + [(\tfrac{R_1}{6} \times 0{\cdot}015625) + (\tfrac{R_2}{6} \times 0{\cdot}001953)]$$
$$= 0{\cdot}001712 R_1 + 0{\cdot}005326 R_2 + 0{\cdot}008637 R_3 + 0{\cdot}004933 R_4$$

When $\gamma = 0{\cdot}5$,

$$\frac{E_c . I_c . I_b . \delta_4'}{(I_c + I_b) L^3} = -(0{\cdot}053527 R_1 + 0{\cdot}090340 R_2 + 0{\cdot}111859 R_3 + 0{\cdot}059469 R_4) \times 0{\cdot}25$$
$$-(0{\cdot}002732 R_1 + 0{\cdot}008030 R_2 + 0{\cdot}012546 R_3 + 0{\cdot}007138 R_4) \times 0{\cdot}026042$$
$$+[0{\cdot}5 \times (R_1 + R_2 + R_3 + 0{\cdot}5 R_4)] \times 0{\cdot}083333 - [0{\cdot}5 \times (0{\cdot}070313 R_1 + 0{\cdot}031250 R_2 + 0{\cdot}007813 R_3)] + [(\tfrac{R_1}{6} \times 0{\cdot}052734) + (\tfrac{R_2}{6} \times 0{\cdot}015625) + (\tfrac{R_3}{6} \times 0{\cdot}001953)]$$
$$= 0{\cdot}001846 R_1 + 0{\cdot}005852 R_2 + 0{\cdot}009794 R_3 + 0{\cdot}005780 R_4$$

When $n = 1$,

$$\frac{L^3 (I_c + I_b)}{I_c . I_b} \cdot (0{\cdot}000628 R_1 + 0{\cdot}001431 R_2 + 0{\cdot}001962 R_3 + 0{\cdot}001073 R_4) = \left(\frac{L^3}{I_c} \times 64{\cdot}4\right) + 255{,}500 - 1{\cdot}1\dot{6} R_1$$

$$\therefore 0{\cdot}000628 R_1 + 0{\cdot}001431 R_2 + 0{\cdot}001962 R_3 + 0{\cdot}001073 R_4$$
$$= \left\{\left(\frac{I_b}{I_b + I_c}\right) \times 64{\cdot}4\right\} + \left\{\frac{I_c . I_b}{L^3 (I_c + I_b)} (255{,}500 - 1{\cdot}1\dot{6} R_1)\right\}$$
$$= \left(\frac{67{,}500 \times 64{\cdot}4}{344{,}000}\right) + \left(\frac{67{,}500 \times 276{,}500}{1728 \times 10^6 \times 344{,}000}\right)(255{,}500 - 1{\cdot}1\dot{6} R_1)$$
$$= 12{\cdot}6366 + 8{\cdot}0227 - 0{\cdot}000037 R_1$$

VI $$\therefore 0{\cdot}000665 R_1 + 0{\cdot}001431 R_2 + 0{\cdot}001962 R_3 + 0{\cdot}001073 R_4 = 20{\cdot}6593$$

Similarly, when $n = 2$,

$$0{\cdot}001307 R_1 + 0{\cdot}003809 R_2 + 0{\cdot}005487 R_3 + 0{\cdot}003040 R_4 = 38{\cdot}6231$$

When $n = 3$,

$$0{\cdot}001712 R_1 + 0{\cdot}005326 R_2 + 0{\cdot}008716 R_3 + 0{\cdot}004933 R_4 = 50{\cdot}1981$$

When $n = 4$,

$$0{\cdot}001846 R_1 + 0{\cdot}005852 R_2 + 0{\cdot}009794 R_3 + 0{\cdot}005864 R_4 = 56{\cdot}1223$$

From these equations,

$R_1 = 35{,}685$ lb; $R_2 = -1{,}517$ lb;

$R_3 = -4{,}360$ lb; $R_4 = 7{,}132$ lb.

VII
$$M'_A = 2400\{(0{\cdot}053527 \times 35{,}685) - (0{\cdot}090340 \times 1{,}517) - (0{\cdot}111859 \times 4{,}360) + (0{\cdot}059469 \times 7{,}132)\} = 4{,}103{,}000 \text{ lb. in.}$$
$$\text{and } H' = \frac{3{,}440}{675}\{(0{\cdot}002732 \times 35{,}685) - (0{\cdot}008030 \times 1{,}517) - (0{\cdot}012546 \times 4{,}360) + (0{\cdot}007138 \times 7{,}132)\} = 415 \text{ lb.}$$

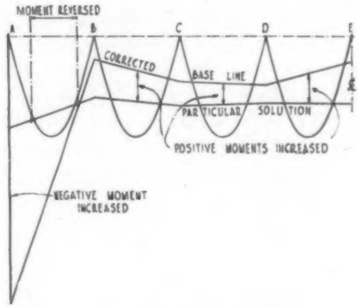


Fig. 9.

The substitution of numerical values is shown in III, the positive numerical value of H' being used.

It may be shown that the increase in the distance between the arch rib and the tie-beam under load (the combined deflections δ' of the arch and tie-beam) is given by

$$\frac{E_c I_c I_b \delta'}{(I_b + I_c)L^3} = \frac{M'_A}{2L}(\gamma^2 - \gamma) + \frac{4hH'}{L} \times \frac{I_b}{I_b + I_c} \times \left(\frac{\gamma^3}{6} - \frac{\gamma^4}{12} - \frac{\gamma}{12}\right)$$
$$- \frac{R}{2}\left(\frac{\gamma^3}{3} - \frac{\gamma}{4}\right) + \frac{R}{6}\left[\gamma - (\tfrac{1}{2} - \alpha)\right]^3 - \frac{R}{2}\alpha^2\gamma.$$

(Negative values of the term $\gamma - (\frac{1}{2} - \alpha)$ are ignored. The symbols are defined in *Fig.* 8.)

If the forces in the tie-rods are $R_1, R_2, R_3 \ldots R_n \ldots R_{\frac{N}{2}}$, this expression may be rewritten as shown in IV. It will be noted that the terms containing γ and α are identical with those in *Fig.* 7, and the substitution of the values relevant to the example are shown in V.

The deflections δ and δ' are evaluated at each tie-rod, and it has already been shown that

$$\delta'_n = \delta_n + \Delta_n - \Delta'_n,$$

in which
$$\Delta_n = \frac{W_n L_r}{A_r E_r} \text{ and } \Delta'_n = \frac{R_n L_r}{A_r E_r}.$$

An estimate of the ratio $\frac{E_r}{E_c}$ must be obtained. Assume that $E_r = 30 \times 10^6$ lb. per square inch, and that the crushing strength of the concrete is 4000 lb. per square inch. Generally E_c may be assumed to be a thousand times the crushing strength,

that is 4×10^6 lb. per square inch. Assuming that A_r is 15 sq. in. and E_r is $7{\cdot}5\ E_c$, the deflections Δ_n and Δ'_n are evaluated in *Table* E, and the substitution in the equation $\delta'_n = \delta_n + \Delta_n - \Delta'_n$ is shown in VI.

R_1, R_2, R_3, and R_4 are the amounts by which the forces in the tie-rods obtained in the particular solution require to be altered. These values are substituted in the expressions for M'_A and H' in VII. The final values are obtained by adding these results to those of the particular solution as in *Tables* F and G.

Similarly the horizontal force in the tie-beam is 887,960 — 415 = 887,545 lb. The moments on the tie-beam are statically determinate, since all the redundant forces and moments are now known. Bending-moment diagrams for the particular solution and the final solution are shown in *Fig.* 9.

The method lends itself to a standard pattern of computation, and the calculation would, of course, be arranged in a more convenient manner in practice. A tabulated solution, for example, reduces the chance of error. It should also be noted that the method can be applied to any arch profile, or to any tie-beam profile, without adding greatly to the labour.

The Thickness of Concrete Road Slabs.

THE Road Research Board has made a further development in the method of determining the thickness of a concrete road slab by setting it into flexural vibrations at frequencies below 500 cycles per second. Measurements were made of the wavelength of the vibrations, and their velocity was computed by multiplying the wavelength by the frequency. To compute the thickness of the slab from these measurements, it was necessary to assume a value of Poisson's ratio for the concrete, but it was found that small changes in Poisson's ratio caused large errors in the computed thickness of the slab.

In the Report of the Board for the year 1955 (H.M. Stationery Office. Price 5*s*.) it is stated that greater accuracy has now been obtained by eliminating the effect of Poisson's ratio within the range of the values of the ratio normally occurring with concrete. The new method depends upon measuring the velocity V_R of Rayleigh, or surface, waves in the concrete at frequencies above about 50,000 cycles per second, as well as measuring the wavelength λ and the velocity V of flexural vibrations set up in the slab at frequencies within the range 1000 to 4000 cycles per second. The Rayleigh waves are independent of the dimensions of the slab but depend on its elastic properties. A curve has been obtained theoretically relating the ratio $\frac{V}{V_R}$ to $\frac{\pi T}{2}$ where T is the thickness of the slab. The thickness T is then found by reading off the value of $\frac{\pi T}{\lambda}$ at measured values of $\frac{V}{V_R}$ and λ. This theoretical curve is not affected by changes in Poisson's ratio within the range 0·20 to 0·25 usually obtained with concrete.

The concrete slab is set into flexural vibration by an electromagnetic generator resting on its surface and operating within the range of 1000 to 4000 cycles per second. The wavelength of the vibrations is determined by measuring the successive distances from the generator at which the spreading vibrations are in phase with those transmitted by the generator, these positions being located by a geophone. When the tests were made at ultrasonic frequencies from 50 to 120 kilocycles per second, barium titanate transmitters and detectors were used. Tests have been made on slabs from 4 in. to 9 in. thick, some on the subgrade and some on prepared bases of soil-cement and hoggin. The results have generally been satisfactory and, in many cases, the dimensions of cores drilled from the slabs have agreed with the calculated values of thickness to within 10 per cent.

Quick Construction with Sliding Shutters.

DURING four days in August last year the concrete walls of a six-story nurses' home, 120 ft. 6 in. long by 36 ft. wide, were constructed at Kirkcaldy by the use of sliding shutters. The building has piled foundations from which the first floor is supported by large reinforced concrete columns. Above this level the structure comprises reinforced concrete external walls, cross-walls, and a spine-wall; the cross-walls are at 17-ft. centres and carry hollow-tile floors and the roof.

From the first floor upwards the walls were cast in continuously-sliding shutters raised by "Prometo" hydraulic jacks. The plan of the walls, all of which were concreted together, is shown in *Fig.* 3.

The shutters were constructed of 4 in.

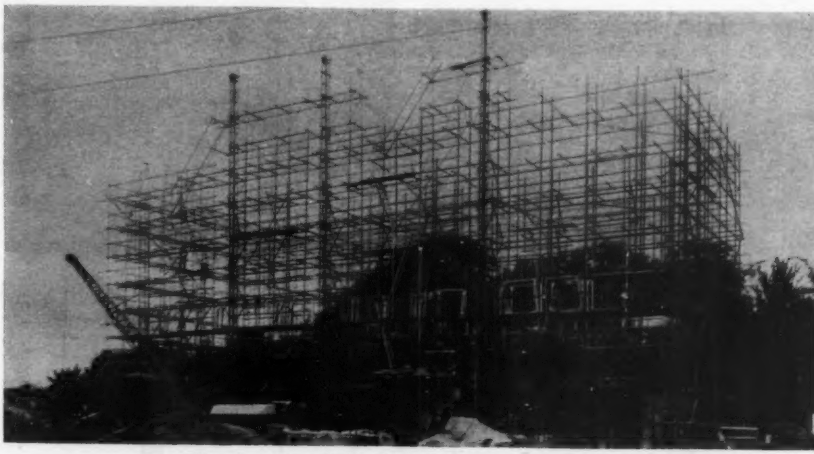


Fig. 1.—Morning of First Day of Concreting.

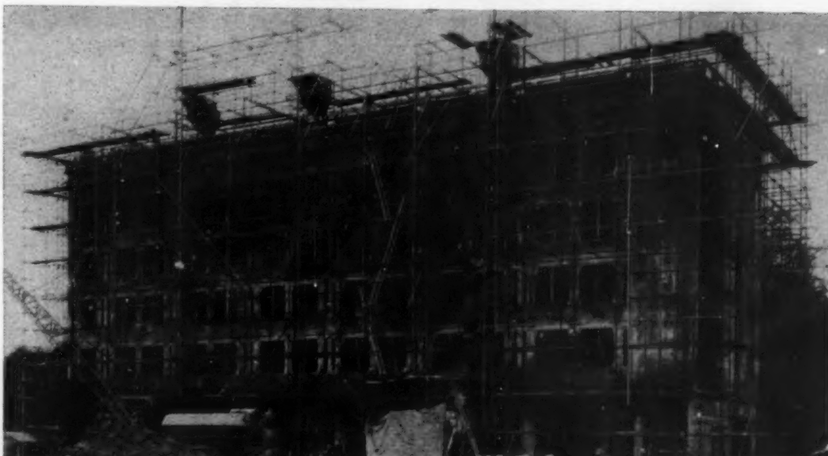


Fig. 2.—Morning of Fourth Day of Concreting.

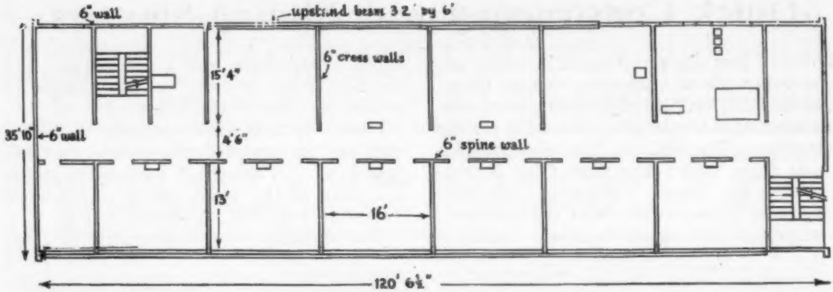


Fig. 3.—Plan of Walls.

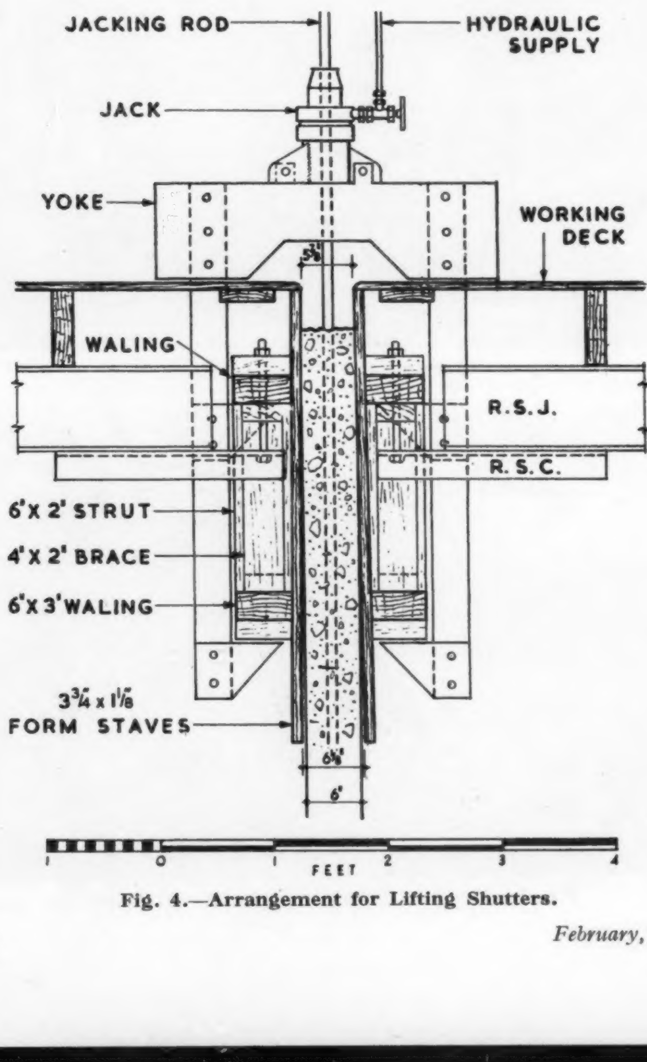


Fig. 4.—Arrangement for Lifting Shutters.

Fig. 5.—Joists and Hanging Scaffold.

by $1\frac{1}{8}$ in. Columbian pine staves and 6 in. by 3 in. walings (*Fig.* 4). They were 4 ft. deep and had a batter of $\frac{1}{8}$ in. except at the outside face of the external walls where they were vertical. *Fig.* 5 shows the shutters during construction, with the joists in position to carry a working deck at the level of the top of the shutters.

A total of 122 jacks were employed, coupled in two circuits to the hydraulic pumps at the control point. Through each jack was placed a high-tensile steel bar of 1 in. diameter with a threaded end for screwing to succeeding lengths. The jacks were connected to standard steel yokes from which brackets projected under the walings of the shutter. At each cycle of operations the jack climbed the steel bar a distance of $\frac{7}{8}$ in., lifting its yoke and the shutter.

It was necessary to provide for window openings in the external walls. Where these were small, and did not coincide with the positions of jacking bars, the openings were formed by inserting a substantial temporary timber frame into the shutter at the required level. To ensure that these frames were set correctly in vertical alignment, dovetailed timber fillets were concreted into the walls at the sides of the openings and the temporary

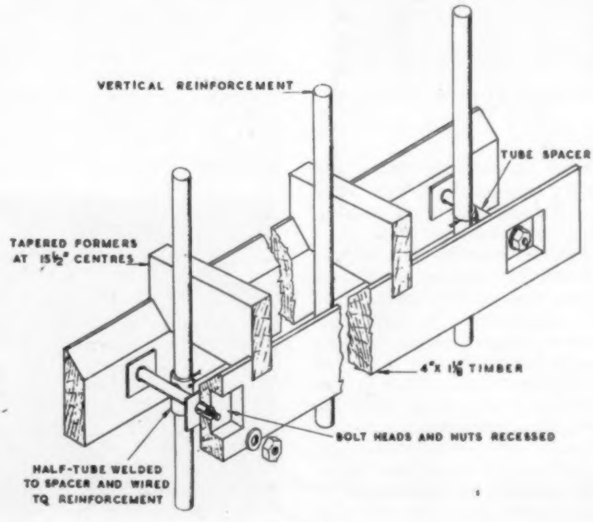


Fig. 6.

frames were nailed to these fillets to prevent any tendency for them to lift with the moving shutters—the fillets were held by concrete which had already hardened. Where a jacking bar passed through an opening for a window a strip of wall 12 in. wide (6 inches on either side of the bar) was concreted. These strips of concrete adequately stiffened the jacking bar and prevented buckling; they were later cut away. A large number of these strips can be seen in *Fig.* 2 prior to being cut away.

It was necessary to make provision in the walls for the floors, which were to be cast later. At the level of each floor pairs of timber battens, bolted together and 3¼ in. apart, were wired to the vertical reinforcement. The purpose of the battens was to form horizontal grooves in the walls to receive the floor slabs. The battens were notched on the top at 15½-in. centres to position timber wedges which

Fig. 7.—Channels formed to receive Wall Ties.

formed the holes through the walls, and through which reinforcement bars were later placed. This arrangement is indicated in *Fig.* 6, and can be seen in place in *Fig.* 8. During construction it was found that the shutter tended to bind on the battens and, despite the wiring, lifted them out of alignment. It was later found easier to cut the necessary grooves and holes by hand immediately the green concrete was exposed; this work was conveniently done from a hanging platform suspended from the moving form.

The gable walls were later to be faced with precast concrete blocks which had to be tied to the concrete, and provision for this was made by concreting into the walls dovetailed metal slots at the required

Fig. 8.—Vertical Reinforcement and Floor Checks in Position.

spacing into which metal ties could later be fixed; these slots are seen in *Fig.* 7.

The concrete materials were batched by weight at two mixing plants at one side of the building. Each plant comprised a 14/10 mixer and a hoist to convey the concrete to a receiving hopper on the working deck. One of these mixers was capable of providing an adequate supply of concrete, but duplication was essential in case of a breakdown. The jacking of the forms proceeded at an average rate of about 7 in. per hour. The weather was reasonably fine, and it was necessary to slow down appreciably only during a heavy fall of rain on the fourth day. *Figs.* 1 and 2 show the building at about 11 a.m. on the first and fourth days.

When roof level was reached the shutters were fixed to the walls and the jacks and yokes dismantled. The roof was then cast, using the working deck as a soffit

Fig. 9.—Underside of Working Platform.

shutter. The casting of the floors then proceeded by ordinary methods, working from the second floor upwards. At the same time, when the roof slab was sufficiently hard, the working deck was lowered, a bay at a time, by hand-operated winches on the roof, to fifth-floor level, where it was supported on timber blocks bolted to the walls and used as shuttering for the hollow-tile floor, and the process repeated. This arrangement is seen from the underside in *Fig.* 9.

As the entire building is to be faced with precast concrete and brickwork the necessary scaffolding was erected to full height before starting to move the sliding shutters. This scaffold was extensively used for placing the wall reinforcement, temporary window frames, and other materials, and also for supporting the flood-lighting equipment required for night work.

The contractors were Messrs. Stuart Construction, Ltd. Messrs. Wm. Thornton & Sons, Ltd., were responsible for providing and supervising the use of the sliding shutters. Mr. J. Holt, F.R.I.B.A., was the architect for the building, which has been built for the South East Scotland Regional Hospital Board. The consulting structural engineers were Messrs. Blyth & Blyth, M.I.C.E., of Edinburgh.

Book Reviews.

"**Architectural Construction — The Choice of Structural Design.**" By Theodore Crane. (London: Chapman & Hall, Ltd. 1956. Price 80*s*.)

ALTHOUGH written primarily for architects in the U.S.A., this book can be confidently recommended to anyone concerned with the structural design of buildings, particularly in the early stages of planning. The book treats each aspect of design in the order in which it would be considered: thus the first chapter briefly reviews current American codes of practice and regulations, and succeeding chapters deal with the choice of materials, the relative merits of the various ways in which the main structural members may be disposed, the construction of floors, roofs, walls, and foundations. Structural steel, reinforced and prestressed concrete, timber, masonry, and brickwork are considered. The scope may be judged from examples ranging from a building of eighteen stories of "flat plate" construction, that is of flat slabs without column capitals, to bungalows with block walls. Although the treatment is based on conditions in the U.S.A., the broad principles are applicable to this country. Precast concrete, particularly prestressed precast concrete, is not emphasised to the extent that it probably would be in a European book on the subject.

The book may be described as a manual on building construction from the poi of view of a structural engineer, not dealing with the dimensioning of members but with the suitability of the possible methods of construction to particular structures, hence such varied topics as weatherproofing, fire-resistance, insulation against heat and sound, and the effects of expansion and contraction due to heat and moisture are given prominence. The volume is well illustrated with photographs and drawings.

"**Einflussflächen für Kreuzwerke.**" By H. Homberg and J. Weinmeister. (Berlin: Springer-Verlag. Second edition. 1956. Price 43.50 D.M.)

THIS book deals with the design of beams for bridges and buildings. These statically-indeterminate systems are commonly met in structural steel design, and to a lesser degree in reinforced and prestressed concrete. In such structures the distribution of loads depends upon the resistance to bending of the transverse girders, on the resistance to rotation of all the members, and upon the resistance of the horizontal slabs and the beams to shearing forces. The authors use accurate methods to assess the resistance to bending of the transverse members, and also deal with the resistance to rotation of the main members. This new edition provides solutions for the usual combinations of freely-supported and continuous beams. Influence areas are given for the static quantities so as to enable systems

that are many times statically-indeterminate to be calculated.

It is claimed that the tables and diagrams allow accurate calculations to be made more quickly than by the usual method. However, it is necessary to use numerous equations and symbols, and these must be mastered in order to apply the tables and diagrams. An introductory chapter provides the necessary explanation. Proofs are not given, but there are references to the literature where these may be found.

"**Curso de Estática das Construções.**" Vol. 1. By Samuel Chamecki. (Rio de Janeiro: Editoria Cientifica. 1956. No price stated.)

THIS is the first volume of a treatise, in the Portuguese language, which the author proposes to complete in four volumes for use in the course on the theory of structures at the University of Paraná. This part deals with general principles and discusses loads, internal and external forces, degrees of redundancy, calculation of forces and bending moments, pressure lines, influence lines, and the calculation of deflections of beams and frames. It concludes with a chapter on factors of safety and the probability of failure. All these subjects are dealt with in about 200 pages, and the consideration given to them, necessarily brief, is that common in Continental rather than in British or U.S.A. books.

"**Resistance of Materials.**" By Fred B. Seely and James O. Smith. (London: Chapman & Hall, Ltd. Fourth edition. Price 52s.)

THIS American book of some 450 pages is an excellent student's text-book written by authors with long experience of teaching technical subjects based on applied mathematics. It deals with the subject in a purely theoretical manner, illustrated with well-chosen examples related to the design of machines and structures, aeroplanes, motor-cars, and so on, so that the student acquires a considerable knowledge of many interesting problems and facts of a very practical kind. The treatment of loads causing buckling of columns, instability, and plastic deformations is a special feature, and gives useful information on unsymmetrical bending (such as commonly occurs in purlins), dynamic effects, and how to adapt structural and machine parts so that they may absorb, without overstress, the greatest possible impact within the capacity of the material. Fatigue of metals is clearly explained and illustrated with examples showing the effects of abrupt changes in sectional area which may cause harmful concentrations of stress. A young man who is versed in this book is well endowed for a career in one of many branches of engineering.—R. P. M.

"**An Introduction to the Theory of Structures.**" By W. Merchant and A. Bolton. (London: Blackie & Son, Ltd. 1956. Price 30s.)

THE scope of this book should be sufficient for the requirements of the first two years of a course in structural engineering. Throughout the authors stress the interdependence of theory and practice, emphasising recent developments so far as is possible in an elementary course. Students will find the book a useful introduction to the subject, particularly as the examples are worked out in detail and there are numerous problems for solution to which answers are given. Brief mention is made of the enveloping curve obtained from Mohr's circles which shows graphically the criterion of strength of a material; this is a useful device infrequently described. In the opinion of the writer more space could usefully have been given to the moment-area method for calculating slope and deflection, and its derivative the conjugate-beam method. The treatment of the latter is more likely to repel the reader than to convince him of its value. The book would be easier to read if the type in which the examples are printed and the diagrams had been larger.

The Analysis of Bridge Decks Subjected to Abnormal Loading. By P. B. Morice and G. Little. [London: Cement & Concrete Association. Gratis.]

A METHOD of analysing the effects on bridges of abnormally heavy loads, using distribution coefficients, is described. The determination of longitudinal and transverse bending moments is considered and it is shown how the torsional strength of the structure can be allowed for in design. A note on the calculation of torsional stiffness emphasises points which must be remembered when applying the method of distribution coefficients.

Precast Corrugated Concrete Roofs.

A PAPER on methods of erecting precast concrete structures presented by Mr. Slavtscho Mirtschev, of Sofia, to a congress held at Dresden, describes the use of corrugated curved concrete slabs for roofs. These roofs are designed as two-pinned arches when the span is less than about 40 ft., and as three-pinned arches for longer spans. The rise is about one-sixth of the span, and the depth of the corrugations varies from about 6 in. to 32 in. according to the span. The width of the slab is such that the weight is suitable for the crane to be used. The thickness of the concrete depends upon the pitch and depth of the corrugations, and is usually from $1\frac{3}{16}$ in. to 2 in.

The materials required, including ties and edge-beams, comprise 12 lb. of steel and the equivalent of a thickness of $1\frac{3}{4}$ in. of concrete per square yard. The joints between adjacent units are at the crests of a corrugation, which improves the watertightness.

A half-section through the roof of a factory is shown in *Fig.* 4. The slabs were precast on the site. The edge-beams of the slabs have a continuous nib which fits into a recess in the supporting beam. The tie-bars pass through the

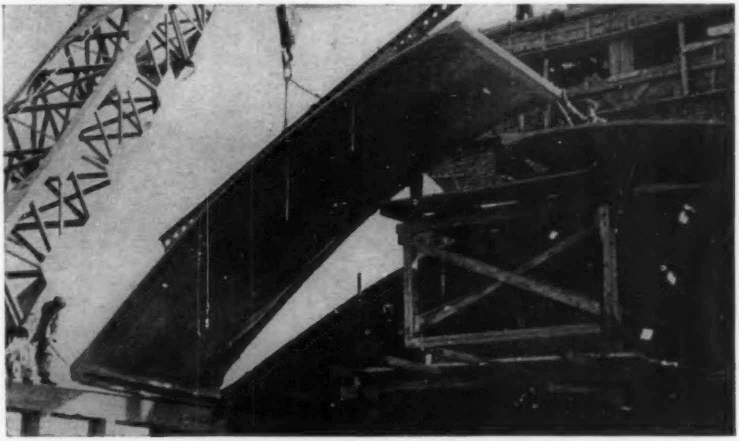


Fig. 1.—Lifting a Slab into Position.

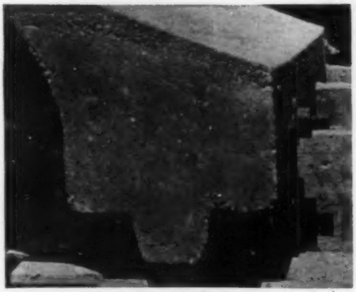


Fig. 2.—Edge-beam and Tie-bar.

Fig. 3.—Joint at Crown.

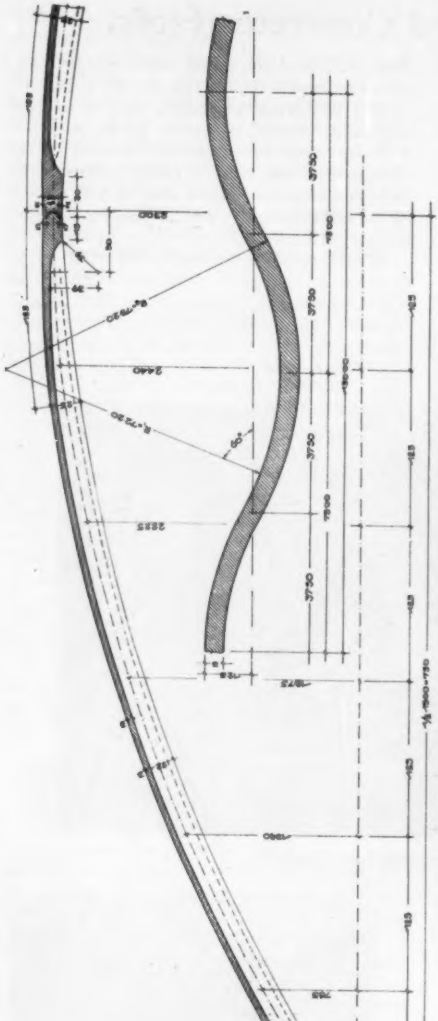


Fig. 4.—Longitudinal and Transverse Sections.

Fig. 5.—Mould for Slabs.

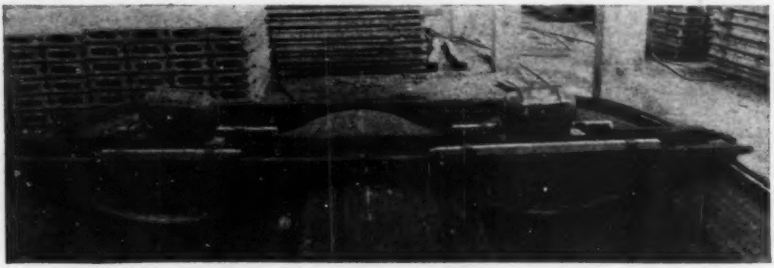


Fig. 6.—Mould and Surface Vibrator.

edge-beam and are anchored by nuts screwed on the ends of the bars and bearing on steel washers (*Fig.* 2). The joint at the crown is formed by a half-round projection on one slab that fits into a recess in the adjacent slab (*Fig.* 3).

A wooden mould for casting slabs is shown in *Fig.* 5. Concrete is distributed evenly in the mould commencing at the crown, and is consolidated by a surface vibrator shaped to give the required contour and supported on rails on the sides of the mould (*Fig.* 6). The edge-beams and crown-beams are consolidated by an immersion vibrator.

A movable frame is generally used to support the slabs at the crown during erection (*Fig.* 1). When the two slabs forming an arch are in position the tie-bars are placed and the supporting frame lowered by screw-jacks until the joint at the crown is closed. The joints at the crown and along the side are then grouted. When subjected to a uniformly-distributed load of 12 tons the factor of safety of a roof similar to that shown in *Fig.* 4 was estimated to be 1·3; the deflection at the crown was $\frac{7}{16}$ in. and at the quarter-points $\frac{1}{4}$ in. No cracks were visible. When the load was removed from half of the arch the factor of safety was calculated to be 3·6, and some cracks were visible in the top of the corrugations of the unloaded part.

Straightening Reinforcement Bars.

A METHOD of straightening bent reinforcement bars is described in a recent summary by the European Productivity agency of an article in the October, 1955, number of the Dutch journal " Vraag en Aanbod ".

For bars of small diameter a steel pipe is bent to the shape shown in *Fig.* 1(a) and hardened. For bars larger than $\frac{1}{2}$ in. diameter a hard steel roller as shown in *Fig.* 1(b) is used; in this case a slot is cut in the tube to enable the roller to bear on the bar and a bridge to hold the bearings of the roller is welded to the tube. The tube is held in the chuck of a lathe with a hollow headstock or in bearings mounted on a frame and is rotated at a high speed while the bars are pushed through the tube.

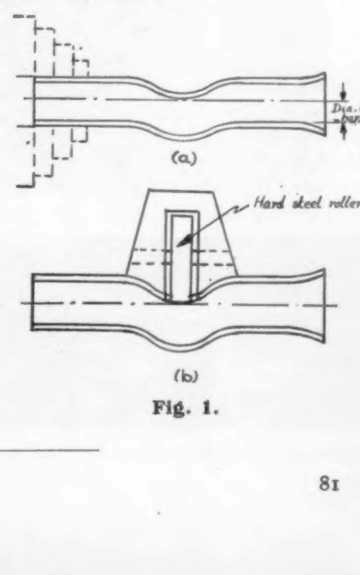


Fig. 1.

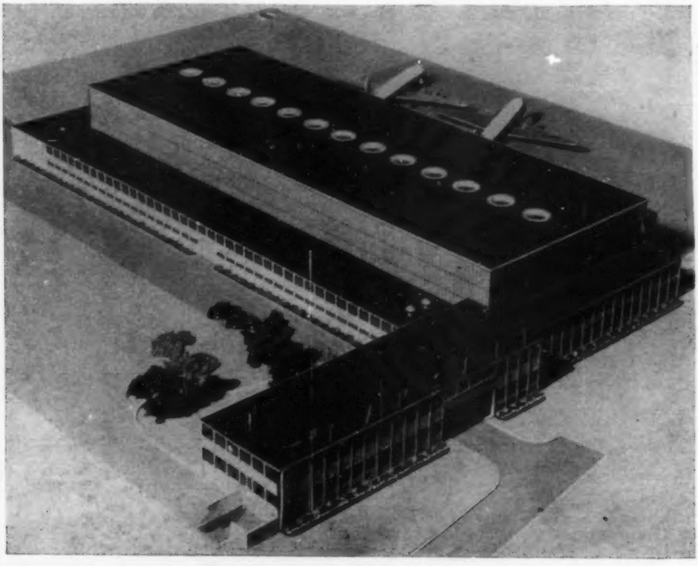
Hangar at Gatwick Airport.

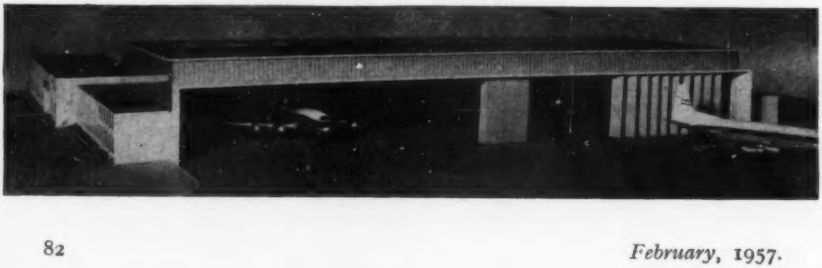
A NEW hangar and auxiliary building with structural frames of precast concrete are to be erected at Gatwick Airport for Transair, Ltd.

The hangar will be 282 ft. long by 112 ft. wide and 41 ft. high ; it will have a clear height of 30 ft. and a maximum clear span of 140 ft. The roof beams will be prestressed. One of the longer sides will have sliding doors ; on the other sides the spaces between the columns will be glazed.

There will be a single-story annex, with a span of 27 ft. 5 in. and a height of 13 ft. 9 in. at one end and at the closed longer side. On the other end there will be a two-story annex 19 ft. wide and 18 ft. 3 in. high. A two-story canteen and office building will project from one end of the longer wall ; this building will have two spans of 19 ft. and a height of 21 ft. The superficial area of the annexes and the canteen and office building will be 24,700 sq. ft., and they will all have walls of glazed panels.

The architects are Messrs. Clive Pascall and Peter Watson. The design of the structural frames is by the London Ferro-Concrete Co., Ltd. (by whom they will be made and erected), in collaboration with Mr. A. J. Harris, B.Sc., M.I.C.E. The contractors are Sir Alfred McAlpine and Son, Ltd.





Vaults Curved in Two Directions.

By S. SIVASWAMI, B.E., A.M.I.E.(India).

LITTLE information seems to be available on the design of vaults curved in two directions, and in the following is presented a method of designing thin slabs corresponding to the short barrel vault but curved in two directions.

Skew surfaces are generated by straight lines but they differ fundamentally from developable surfaces such as a cylinder or cone in which a single tangent plane touches the surface at all points along each generating line and the normals PN (*Fig.* 1) at points along a generating line are all parallel. The normals along AB and A'B' intersect, and it is possible to rotate A'B' about AB so that PN and P'N' become parallel, in which case the surface becomes a plane. However, in a skew surface (*Fig.* 2) the normals do not have a fixed direction but radiate from the generator. *Fig.* 2 illustrates the difference between the generators of

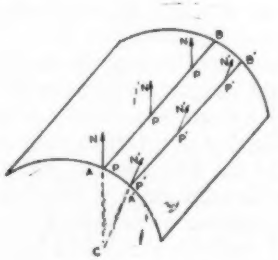


Fig. 1.—A Developable Surface.

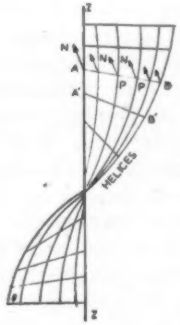


Fig. 2.—A Skew Surface (Helix-conoid).

a developable surface and a skew surface. As the normals of a skew surface are neither parallel nor intersecting they are non-coplanar.

The membrane theory is based upon the development of hoop stresses to resist the normal forces. For the membrane to act as a single element, consecutive normals must meet. Lines drawn along a surface such that the consecutive normals intersect are called lines of curvature, and the normal sections at any point through the tangent to these lines are principal sections. At every point on a surface there will be two principal sections, which will be at right angles to each other. In the case of a developable surface the generating lines are lines of curvature, but in the case of skew surfaces they are not. Every membrane other than a catenary-cylinder has to be reinforced.

The load will be distributed in the ratio of the stiffness along the two principal sections. In a balanced design the membrane will be equally strong in tension and compression. Hence it can be assumed that the normal load is equally distributed in the two principal sections. The hoop stress developed will be $-\rho Z$ with the proper sign. Curvature is assumed to be positive if the centre of the curvature is below the surface, and Z (the load) is positive downwards.

If the equation of the surface is

$$z = f(x, y) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, and

$$k = \sqrt{1 + p^2 + q^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the principal radii of curvature are given by the quadratic equation

$$(rt - s^2)\rho^2 - k[(1 + p^2)t + (1 + q^2)r - 2spq]\rho + k^4 = 0. \quad . \quad (3)$$

The projections of the tangents to lines of curvature on the plane $z = 0$ are inclined at an angle α such that $\tan \alpha = \frac{dy}{dx}$ where $\frac{dy}{dx}$ is given by

$$[pqt - s(1 + q^2)]\left(\frac{dy}{dx}\right)^2 + [(1 + p^2)t - (1 + q^2)r]\frac{dy}{dx} - [pqr - (1 + p^2)s] = 0 \quad (4)$$

The principal radii of curvature of (1) a rectangular hyperbolic paraboloid, (2) right conoids, (3) a right helicoid, and (4) a single-sheet hyperboloid of revolution will be evaluated.

The hyperbolic paraboloid is the simplest, and it has the fundamental importance in the theory of skew surfaces that the tangent plane has in the theory of developable surfaces. It is possible to find hyperbolic paraboloids to touch any skew surface along a generator. In the following its principal radii and lines of curvature are derived by two methods. In the first method the principal radii of curvature are expressed in terms of the co-ordinates x, y, z of every point on it, and in the second method the radii of curvature are worked out along each generator of the hyperbolic paraboloid. The hyperbolic paraboloid is a surface generated by a straight line cutting two non-intersecting lines and moving parallel to a given plane (*Fig.* 3). Where the plane is perpendicular to one of the non-intersecting straight lines a rectangular hyperbolic paraboloid is obtained.

METHOD 1.—With the plane as the xz plane, the perpendicular straight line as the axis OY, and the common perpendicular between the two straight lines as the X axis, the simplest equation of a rectangular hyperboloid is

$$z = mxy \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

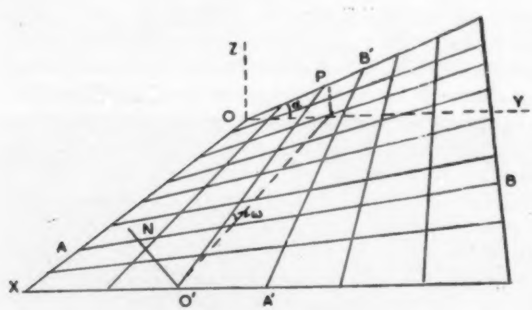


Fig. 3.—Hyperbolic Paraboloid.

In the surface illustrated in *Fig.* 3 the planes $x = A$ and $y = B$ cut the surface along the generators.

$$p = my,\ q = mx,\ r = 0,\ s = m,\ t = 0.$$

Therefore $-\ s^2\rho^2 + 2ks\rho pq + k^4 = 0$

and $$\rho = \frac{k}{s}\{pq \pm \sqrt{k^2 + p^2q^2}\} = k\left\{z \pm \sqrt{\frac{1}{m^2} + x^2 + y^2 + z^2}\right\} \quad . \quad (6)$$

Equation (4) becomes $dx^2(1 + p^2) = dy^2(1 + q^2)$.

Therefore $\dfrac{dy}{\sqrt{1 + m^2y^2}} = \pm \dfrac{dx}{\sqrt{1 + m^2x^2}}$; $\sinh^{-1} mx = \pm \sinh^{-1} my$.

The real root is $$x = \pm y \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

that is, the projections of the lines of curvature on the plane $z = 0$ are inclined at 45 deg. to the co-ordinate axes.

METHOD 2.—Let $P(x_1y_1z_1)$ be any point on the paraboloid and let O′P be the generator through P (*Fig.* 3) meeting OY at the centre point O′. Transferring the origin to O′ and rotating the axes of co-ordinates about OY until the X axis coincides with O′X and the Z axis with the normal O′N,

$$z' = x \sin \omega + z \cos \omega,\ x' = x \cos \omega - z \sin \omega,$$
$$y' = y + y_1,\ \text{and}\ z(x_1 + my_1z_1) = my(xx_1 - zz_1).$$

Writing $m' = \dfrac{m}{x_1 + my_1z_1}$, the equation becomes

$$z = m'y(xx_1 - zz_1) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\left.\begin{aligned} p &= \frac{m'x_1y}{1 + m'z_1y},\ q = \frac{m'(xx_1 - zz_1)}{1 + m'z_1y},\ r = 0 \\ s &= \frac{m'x_1(1 + m'z_1y) - m'^2x_1z_1y}{(1 + m'z_1y)^2},\ t = \frac{-\ 2m'^2(xx_1 - zz_1)z_1}{(1 + m'z_1y)^2} \end{aligned}\right\} \quad . \quad (9)$$

Any point P on the generator is given by $(x\ 0,\ 0)$. Therefore along the generator

$$p = 0;\ q = m'xx_1;\ r = 0;\ s = m'x_1;\ t = -\ 2m'^2x_1z_1x;\ k^2 = 1 + q^2.$$

Substituting in equation (3), $\dfrac{s^2\rho^2}{k^2} + t\dfrac{\rho}{k} - (1 + q^2) = 0.$

Therefore $2s^2\dfrac{\rho}{k} = -\ t \pm \sqrt{t^2 + 4s^2(1 + q^2)}$

$$= 2m'z_1xs \pm \sqrt{4s^2m'^2z_1{}^2x^2 + 4s^2(1 + m'^2x^2x_1{}^2)}.$$

Therefore $$\frac{\rho}{k} = \frac{1}{m'x_1}\{m'z_1x + \sqrt{m'^2z_1{}^2x^2 + 1 + m'^2x^2x_1{}^2}\} \quad . \quad . \quad (10)$$

Similarly the angle α made by the lines of curvature with the axis X is given by $\tan \alpha = \dfrac{dy}{dx}$,

$$= \frac{-\ m'z_1x \pm \sqrt{1 + m'^2x_1{}^2x^2 + m'^2x^2z_1{}^2}}{1 + m'^2x^2x_1{}^2} \quad . \quad . \quad . \quad (11)$$

The first method is simpler and is applicable to the case of a hyperbolic paraboloid. The second method is applicable to all skew surfaces that are touched by a hyperbolic paraboloid along a common generator.

Surfaces known as conoids are generated by a straight line that moves parallel to a given plane and intersects a given curve and a given straight line not coplanar with the curve. The elements associated with the formation of a conoid are (1) the generating curve APB, (2) the axis XX, (3) the generating straight lines PQ′, and (4) the plane to which the straight lines are parallel (ZX). In the right conoid the axis is perpendicular to the plane. The generating curve may be either a plane curve or a skew curve as shown in *Figs.* 4 and 2.

Conoidal-shaped roofs usually have plane generating curves only, but the method developed here is applicable to all conoids. The hyperbolic paraboloid is a particular case of a conoid in which the generating curve is a straight line. As the generating curve is in fact the envelope of its tangent, considered as moving according to a given law, the conoid is the envelope of a hyperbolic paraboloid which moves according to a related law.

The method of obtaining the simplest equation of the hyperbolic paraboloid has been explained. The axes are the straight line OY perpendicular to the generating plane, the straight line OX being the common perpendicular of O′A′ and OY and OZ perpendicular to both OX and OY. Hence in elevation the projections of OA and OY intersect at the projection of O. The angle YOB′ is constant and equal to, say, α.

In the case of a conoid the angle θ is variable, corresponding to any point P on the curve. The tangent PT, and hence the position of T, vary. Referred to axes TY, TZ and TX, the equation of the hyperbolic paraboloid touching the conoid along a generator can be expressed in the simple form $z = mxy$. As T is now variable, m is variable. We are thus expressing the equation of the conoid with reference to moving axes. The radii and lines of curvature of the moving hyperbolic paraboloid may now be calculated by Method 2 for the generator PQ and the same values are applicable to the conoid.

The helicoid is a particular case of a conoid with a skew generating curve, namely the right circular helix. The method outlined for a conoid is applicable in this case; here, however, the generating hyperbolic paraboloid is constant but rotates about the axis as it moves along the axis. The equation of the helicoid is so simple that its radii of curvature can be derived directly. If α is the constant inclination of the tangent to the arc of the generating cylinder of radius a,

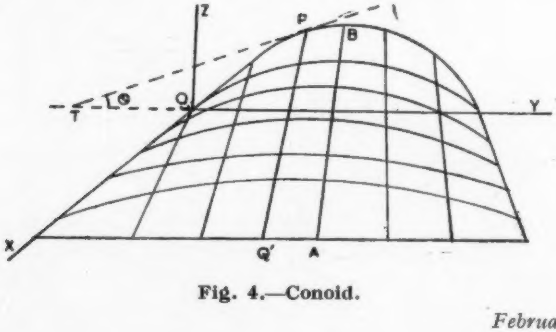


Fig. 4.—Conoid.

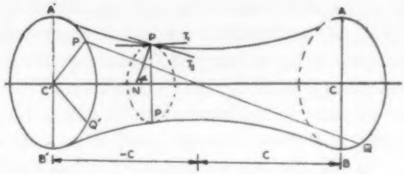


Fig. 5.—Single Sheet Hyperboloid.

$$z = a \tan \alpha . \theta . \quad x = u \cos \theta . \quad y = u \sin \theta \quad . \quad . \quad . \quad (12)$$

where u is the cylindrical radius vector of any point on the helix.

Writing $c = a \tan \alpha$,

$$p = \frac{-cy}{u^2},\ q = \frac{cx}{u^2},\ r = -t = \frac{2cxy}{u^4},\ s = \frac{c(y^2 - x^2)}{u^4},\ k^2 = \frac{u^2 + c^2}{u^2} \quad . \quad (13)$$

If u and θ are the polar co-ordinates of the projections of the curves on the plane $z = o$,

$$\rho = \pm \frac{u^2 + c^2}{c} \text{ and } \tan \phi = \frac{u d\theta}{du} = \pm \frac{u}{\sqrt{u^2 + c^2}}$$

where ϕ is the angle of inclination of the projection of the tangent to the lines of curvature to the radius u.

The single-sheet hyperboloid of revolution is a skew surface of a different class (*Fig.* 5). Assuming that the axis of revolution is the axis z and the origin is at the narrow section of the surface, OX and OY are any two perpendicular axes. Let A'PB', AQB be any two sections, $z = \pm c$, equidistant from the plane $z = 0$. The generators are lines such as PQ joining conjugate diameters of the equal circles A'PB', AQB. Obviously, the shape of the element of the surface in the neighbourhood of PQ is the hyperbolic paraboloid PP'T_1, the generating plane being inclined at 45 deg. to the radii CP and CQ. Thus the single-sheet hyperboloid is also easily reduced to a hyperbolic paraboloid revolving about the axis z. The single-sheet hyperboloid of revolution can more easily be dealt with directly, as it is a surface of revolution generated by a revolving hyperbola. Its equation is of the form

$$\frac{u^2}{a^2} - \frac{z^2}{b^2} = 1 \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The lines of curvature through any point P are the hyperbola APB and the circular section PQR through P. If rko is the radius of the circular section and α the angle of inclination of the normal at P to the axis z, the principal radius of curvature

$$\rho_1 = PN = \rho \sec \alpha \text{ (by Meunier's theorem)} \quad . \quad . \quad . \quad (15)$$

The other principal radius is that of the curvature of the hyperbola at P (U, z).

$$\rho_2 = \frac{-\,(\text{normal})^3}{(\text{semi-latus rectum})^2} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

It is thus clear that the hyperbolic paraboloid plays a vital role in the membrane theory.

Having found the radii of curvature ρ_1 and ρ_2 and the tangents to the lines of curvature PT_1 and PT_2 at any point P at which the normal is PN, the weight acting on unit area surrounding P may be resolved into three mutually perpendicular components along PN, PT_1, and PT_2 (*Fig.* 6). The component along the normal PN is resisted equally by hoop stresses along PT_1 and PT_2, which act as direct stresses whereas the components along PT_1 and PT_2 act as shearing forces on the element. If the direction cosines of PZ with reference to PN, PT_1, PT_2 are cos α, cos β, and cos γ respectively, and if W_0 is the load per square foot at P,

$$\left.\begin{aligned}\text{Force along PN} &= W_0 \cos\alpha = N\\ \text{Force along PT}_1 &= W_0\cos\beta = X\\ \text{Force along PT}_2 &= W_0\cos\gamma = Y\end{aligned}\right\} \quad . \quad . \quad . \quad (17)$$

Considering the element $dx.dy$ with reference to the lines of curvature that form a system of orthogonal curvilinear co-ordinates on the surface,

$$T_1 = -\tfrac{1}{2}W_0\cos\alpha\rho_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$T_2 = +\tfrac{1}{2}W_0\cos\alpha\rho_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

T_1 and T_2 are of opposite signs as ρ_1 and ρ_2 are of opposite signs.

For the equilibrium of the element (*Fig.* 7),

$$\frac{\partial S}{\partial y} - \frac{\partial T_1}{\partial x} + X = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$\frac{\partial S}{\partial x} + \frac{\partial T_2}{\delta y} + Y = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

For equations (1) and (2) to be consistent

$$S = \int\left(\frac{\partial T_1}{\partial x} - X\right)dy = -\int\left(\frac{\partial T_2}{\partial y} + Y\right)dx \quad . \quad . \quad . \quad (22)$$

In general (22) will not be satisfied unless $\rho_1 = \rho_2$ and $\beta = \gamma = 45$ deg. (This is the case at every point in a helicoid.) This discrepancy is due to the fact that the displacements assumed are not consistent.* For all practical purposes we may assume for S the higher of the two values given by equations (20) and (21). Where the values of S so obtained differ considerably, it cannot be assumed that the normal traction is equally resisted by the hoop stresses along the principal sections; it may be necessary to distribute them in a certain ratio (say 1 : K) such that the two shearing forces are equal.

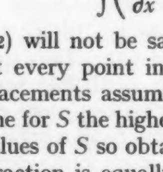


Fig. 6.

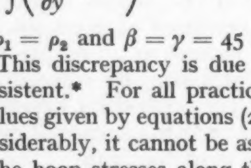


Fig. 7.

* A similar discrepancy has been noticed even in the more general case of the deformation of surfaces assumed by A. E. H. Love. "Mathematical Theory of Elasticity," article 336, page 565.

It is found that in the types of vaults considered $\rho_1 - \rho_2$ will be about 2 to 3 per cent. only and the angles β and γ differ only by 3 or 4 deg. from 45 deg. Hence the discrepancy between the two values of S will be very small.

The values of θ_1 and θ_2 obtained from equation (4) enable β and γ to be found. The angle θ_1 is the angle made by the projection pt_1 of PT_1 on the plane XY with the axis X, and similarly θ_2 is the angle between the projection pt_2 of PT_2 on the plane XY and the axis X.

The equations of pt_1 and pt_2 on the plane XY are therefore $y = x \tan \theta_1$ and $y = x \tan \theta_2$. PT_1 and PT_2 are the lines of intersections of the planes $y = x \tan \theta_1$ and $y = x \tan \theta_2$ with the tangent plane to the given surface at the point P. If the equation of the curve is expressed in the form $z = f(x, y)$, and if $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$, the equation of the tangent plane T_1PT_2 is $z = px + qy + (z_1 - px_1 - qy_1)$.

The equations of PT_1 are $y = x \tan \theta$; $z = px + qy + (z_1 - px_1 - qy)$.

The equations of a parallel straight line through the origin are $y = x \tan \theta_1$; $z = px + qy$; that is $\dfrac{x}{-1} = \dfrac{y}{-\tan \theta} = \dfrac{z}{-p - q \tan \theta}$. Therefore the direction cosines of PT_1 are

$$\frac{1}{\sqrt{1 + \tan^2 \theta_1 + (p + q \tan \theta_1)^2}}, \quad \frac{\tan \theta_1}{\sqrt{1 + \tan^2 \theta_1 + (p + q \tan \theta_1)^2}},$$
$$\frac{p + q \tan \theta_1}{\sqrt{1 + \tan^2 \theta_1 + (p + q \tan \theta_1)^2}}.$$

Similarly the direction cosines of PT_2 are obtained by writing $\theta_2 = \theta_1$ in the foregoing.

The direction cosines of the normal PN are

$$\frac{-p}{\sqrt{1 + p^2 + q^2}}, \quad \frac{-q}{\sqrt{1 + p^2 + q^2}}, \quad \frac{1}{\sqrt{1 + p^2 + q^2}}.$$

The direction cosines of the vertical through P with reference to PT_1, PT_2 and PN are therefore

$$\cos \beta = \frac{p + q \tan \theta_1}{\sqrt{1 + \tan^2 \theta_1 + (p + q \tan \theta_1)^2}}; \quad \cos \gamma = \frac{p + q \tan \theta_2}{\sqrt{1 + \tan^2 \theta_2 + (p + q \tan \theta_2)^2}};$$

$$\cos \alpha = \frac{1}{\sqrt{1 + p^2 + q^2}}.$$

Nuclear Congress in the U.S.A.

A Nuclear Congress, sponsored by more than twenty U.S.A. engineering and technical societies, is to be held in Philadelphia from March 11 to 15, 1957. An International Atomic Exposition on the use of nuclear energy for civil purposes has been arranged in connection with the congress. Papers on the structural design of reactor installations and on structural protection against, and control of, fission products are included among the 200 technical papers to be presented. Further information may be obtained from Engineers Joint Council, 29 West 39th Street, New York 18, N.Y., U.S.A.

A Sports Arena in Portugal.

In November, 1951, the Municipality of Oporto decided to build an indoor sports arena. As the World Roller-skating Hockey Championship was to be held there the following July the work, including the demolition of an old building, had to be done quickly, and the structure designed as the work progressed.

The Palacio dos Desportos (*Fig.* 1) is a circular building of 92 m. (302 ft.) diameter. The main floor can be adapted to accommodate 10,000 people. The central part of this floor is supported on columns and the outer circular part, 10 m. (33 ft.) wide, is supported by 32 triangular frames (*Fig.* 2). These frames also support the three galleries encircling the building. The upper vertices of the triangular frames form a circle 72 m. (236 ft.) diameter, and support a concrete dome formed by thirty-two meridian ribs and eight ring-beams, between which are concrete slabs with circular glass lights. The opening at the apex of the dome is covered by a smaller dome of reinforced concrete and glass, providing light and ventilation. The summit of the dome is accessible by means of a helical staircase. In the basement, covering an area of 6400 sq. m. (69,000 sq. ft.), are the offices, restaurant, bar, cloak-room, telephone booths, and so forth.

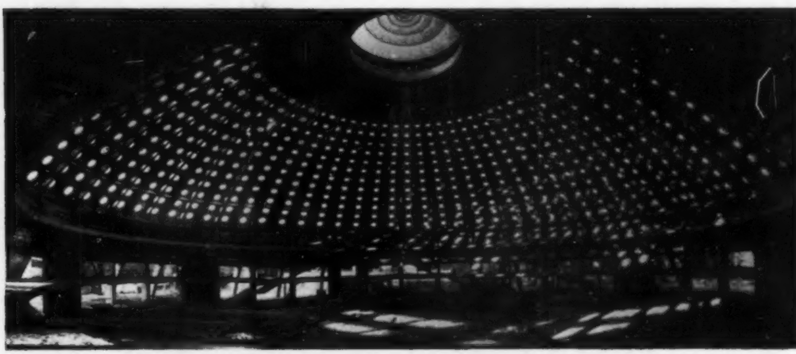


Fig. 1.—Sports Arena at Oporto.

The columns below the central part of the main floor support beams which intersect at right angles, forming a grid 7 m. by 6 m. (23 ft. by 20 ft.). The rectangles so formed are filled with prestressed concrete slabs.

The main supporting structure comprises the thirty-two triangular frames and is divided into eight segments by contraction joints. The frames are 16 m. (52 ft. 6 in.) high and 10 m. (33 ft.) across the base. The outer legs follow the curves of the meridional arch ribs. The inner leg is slightly inclined and carries the main floor and the three galleries. The loads carried by the triangular frames are mainly from the dome.

The dome is 72 m. (236 ft.) diameter and has a rise of 15 m. (49 ft.) and a radius of curvature of 48 m. (158 ft.). The meridional ribs transmit the lateral thrust directly to the triangular frames without an edge-beam. At first, the ribs and frames were joined by means of temporary hinges. A year after the removal of the shutters, when most of the contraction had taken place, the hinges were fixed, enabling bending moments to be resisted. The dome is covered on the outside with concrete made with cork aggregate and inside with a sound-absorbing material.

Building began on February 10, 1952. In four months the building was erected up to the supports of the dome, concrete being placed at an average rate of 50 cu. m. (65 cu. yd.) a day, and after the skating championship had taken place the dome was completed. Concreting commenced at the upper ring, followed by the various meridians and ring-beams and

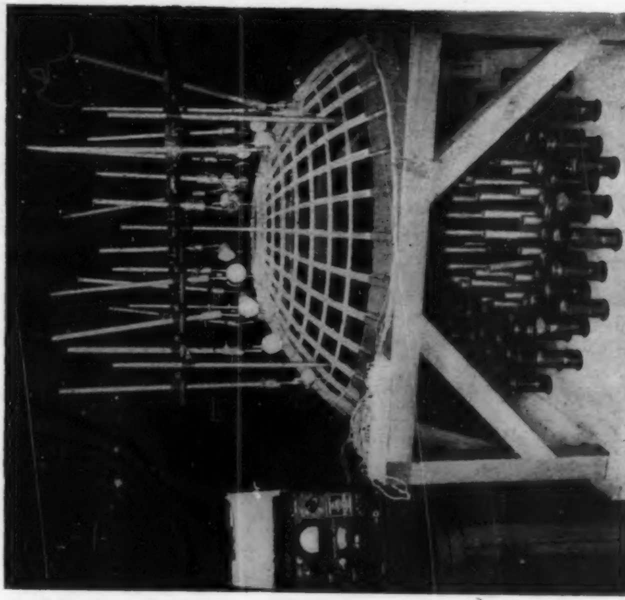


Fig. 3.—Test of a Model.

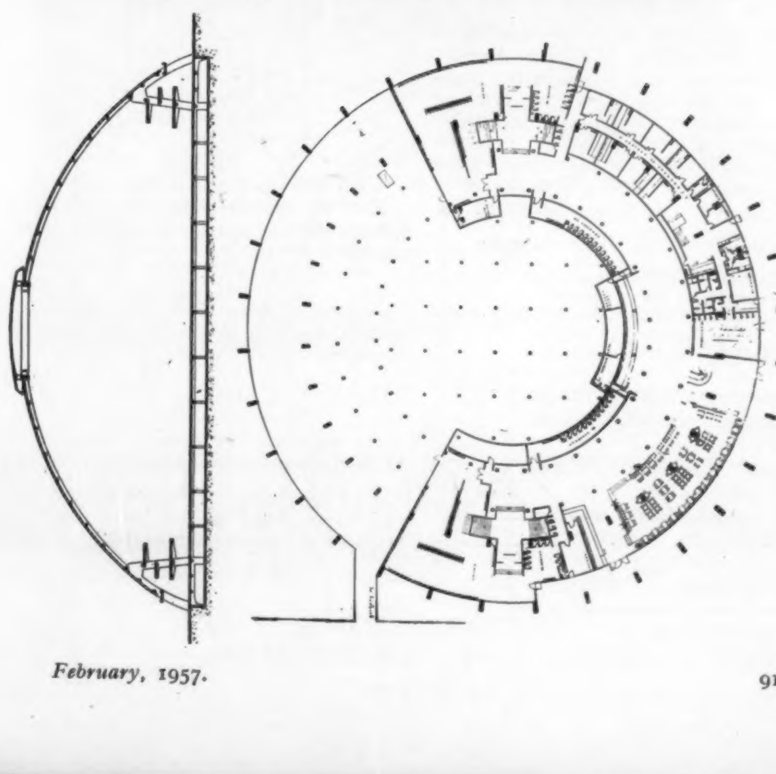


Fig. 2.—Plan and Section.

finally by the panels between the ribs. In the meridians there were two concreting joints in order to prevent stresses being produced by settlement of the timber shuttering under the weight of concrete.

Tests were made on a model (*Fig.* 3) at the National Laboratory of Civil Engineering to estimate the behaviour of the structure when subjected to various stresses and concentrated loads. About thirty vibrating charges were placed, which enabled stresses of 0·6 kg. per square centimetre (8 lb. per square inch) to be measured. Measurements were also made of the variations of the opening of the joints joining the upper vertices of the triangular frames, and vertical and radial displacement of certain points. A perspex model was built to a scale of 1 : 75 for testing under similar conditions, and electric extensometers were placed at points corresponding to those where the charges had been placed in the large model.

The architect is Sr. J. Carlos Loureiro. Sr. António A. Santos Soares is the engineer, and Sr. A. Ramalheira the contractor.

Edge-beams of Slabs Spanning in Two Directions.

WE have received the following note from Mr. P. Cohen, B.Sc., of Johannesburg, on a method of calculating the maximum bending moments and shearing forces on the beams along the edges of rectangular slabs spanning in two directions and supporting a uniformly-distributed load. The method is based on the B.S. Code of Practice No. 114 (1948), Clause 309*c* (iv) (1).

Notation.—w is the equivalent load per foot, that is the constant uniformly-distributed load that produces the same maximum shearing force or bending moment as is produced by the triangular or trapezoidal load in *Fig.* 1. p is the total uniformly-distributed load per square foot of slab. L_x is the length of the shorter beams in feet. L_y is the length of the longer beams in feet. F is the ratio $\frac{L_x}{L_y}$. J is the shearing constant $\frac{2 - F}{4}$. K is the bending moment constant $\frac{3 - F^2}{6}$.

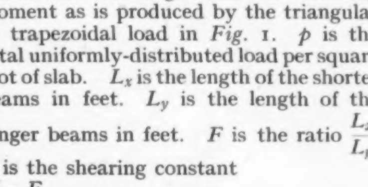


Fig. 1.

TABLE I.

F	J	K
0·0	0·50	0·50
0·1	0·48	0·50
0·2	0·45	0·49
0·3	0·43	0·49
0·4	0·40	0·48
0·5	0·38	0·46
0·6	0·35	0·44
0·7	0·33	0·42
0·8	0·30	0·40
0·9	0·28	0·37
1·0	0·25	0·34

Table I gives J and K for values of F.

For the shorter beam the equivalent load (w) to produce the maximum shearing force is

$$\frac{pL_x}{4} \text{ lb. per ft.,}$$

and to produce the maximum bending moment

$$\frac{pL_x}{3} \text{ lb. per ft.}$$

For the longer beam the equivalent load to produce maximum shearing force is

$$JpL_x \text{ lb. per ft.,}$$

and to produce the maximum bending moment the equivalent load is

$$KpL_x \text{ lb. per ft.}$$

Translucent Dome of 31 ft. Diameter.

THE translucent dome shown in the illustrations forms the roof of the music room at Netteswell Modern Grammar School, Harlow. The roof (*Figs.* 1 and 2) is a segment of a sphere of 31 ft. 2 in. diameter centre to centre of bearings; the overall thickness is 2⅜ in. nominal.

Toughened glass lenses of 4⅜ in. nominal diameter and 2⅜ in. deep were placed on the centering at 10½ in. centres, and the spaces between them filled with concrete in the proportions of 1 part rapid-hardening Portland cement, 1½ parts graded river sand, and 3 parts graded granite chippings with a maximum size of ⅜ in. The water-cement ratio did not exceed 0·5. This mixture produces a dense impervious concrete with a compressive strength exceeding 5000 lb. per square inch at seven days. The elastic modulus of the concrete is between 3×10^6 and $3{\cdot}5 \times 10^6$ lb. per square inch, and that of the glass lenses is between 10×10^6 and $10{\cdot}7 \times 10^6$, resulting in a modular ratio of about 3 : 1 between the glass and the concrete. The concrete has a coefficient of thermal expansion of about 5×10^{-6} per deg. F.; this is almost the same as that of the glass so that changes of temperature have the same effect on both materials.

The centering was lined with hardboard, upon which the lenses were set before the reinforcement was placed. The area of the roof is 881 sq. ft., and concreting was carried out continuously in order to avoid construction joints. The soffit was sprayed with ⅜-in. thickness of white asbestos fibre for acoustical purposes.

The roof was designed as a " flat dome ", that is assuming that no hoop tension would be present. The radial thrust is resisted by a peripheral tensile band at the springing (*Fig.* 3). The only superimposed load provided for was 15 lb. per square foot for snow. Originally it was intended to construct the dome together with the circular lintel and cantilevered

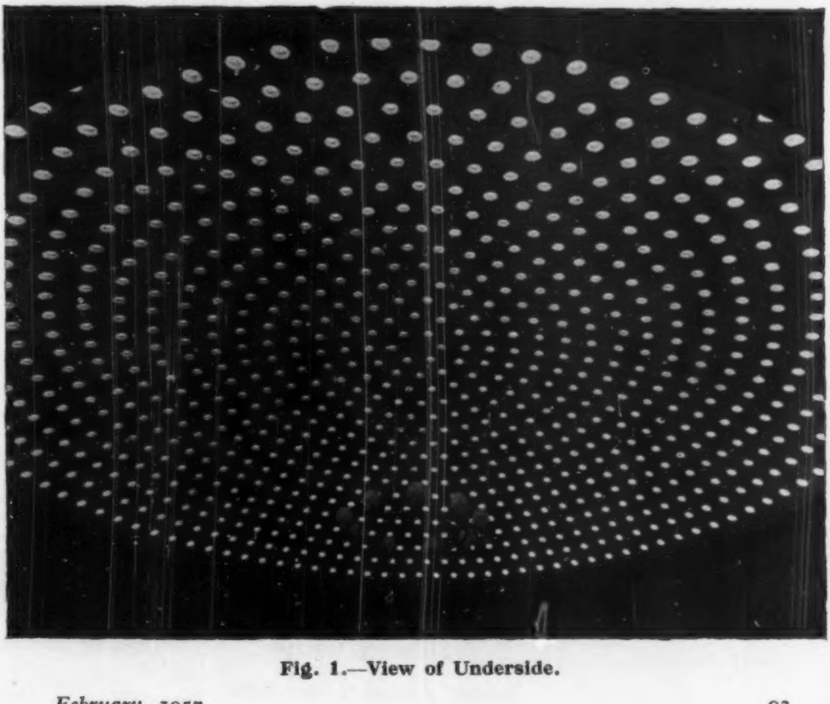


Fig. 1.—View of Underside.

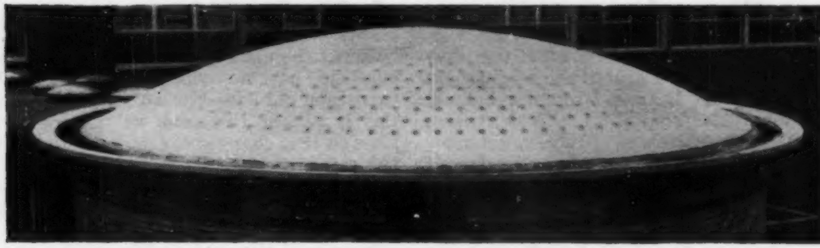


Fig. 2.

canopy, but this idea was abandoned so as to avoid a reversal of stress near the periphery with the consequent risk of tensile cracks in the upper surface.

Freedom of movement at the bearing is achieved by lining the rebate with bituminous felt and filling with an expansion jointing material. The joint was weatherproofed by turning the asphalt from the canopy into a rebate around the perimeter. The secondary stresses together with the effects of wind and suction over parts of the surface were indeterminate, and it was chiefly for this reason that toughened lenses were used; during manufacture these lenses are thermally case-hardened and will withstand a tensile stress of 20,000 lb. per square inch so that they can resist reversals of stress of short duration.

The architects for the school are Messrs. Hening & Chitty, in conjunction with the County Architect of Essex. The consulting engineers are Messrs. Andrews, Kent & Stone. The dome was designed and constructed by Lenscrete, Ltd., of London.

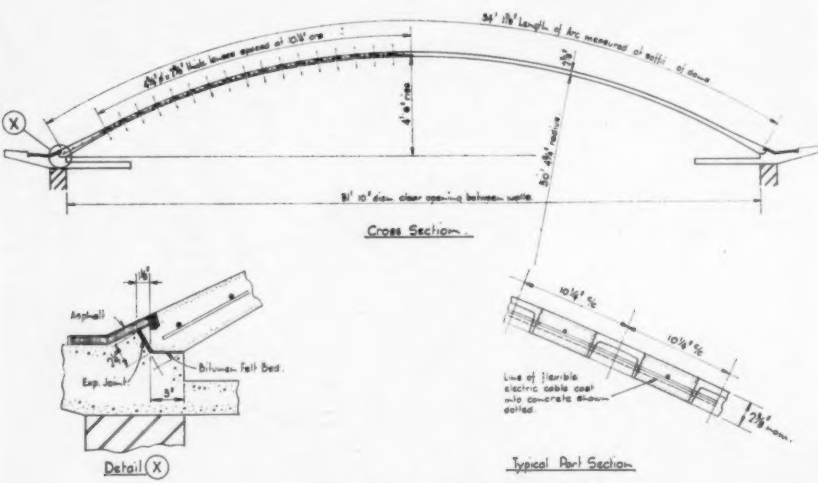


Fig. 3.—Details of Dome.

Training Courses on Concrete.

DURING the months of April to December (except August) the popular training courses arranged by the Cement and Concrete Association will be held at the Association's training centre at Wexham Springs, near Slough, Buckinghamshire. The total cost of a course of 4½ days does not exceed £10, including board and lodging at the Association's hostel. The courses are as follows.

April 2 to 4: Road and bridge design and construction (for chief technical officers of local authorities).

April 8 to 12, April 29 to May 3, and May 6 to 10: Structural concrete (for supervisors).

May 20 to 24, May 27 to 31, and June 3 to 7: Structural concrete (for engineers).

June 17 to 21: Concrete road and soil-cement construction (for supervisors).

July 1 to 4: Soil-cement construction (for engineers).

July 15 to 19: Concrete road construction (for engineers).

September 9 to 13: Concrete construction (for builders).

September 16 to 27: Course for overseas engineers in conjunction with the British Council.

October 7 to 11, October 14 to 18, and October 21 to 25: Properties of concrete and control of quality (for engineers).

November 4 to 8: Concrete products (for members and employees of firms subscribing to the Research Committee for the Cast Stone and Cast Concrete Products Industry).

November 11 to 15: Design of pre-stressed and reinforced concrete bridges (for engineers).

November 18 to 22, November 25 to 29, and December 2 to 6: Advanced structural concrete (for engineers).

Full details are obtainable from the Cement and Concrete Association, 52 Grosvenor Gardens, London, S.W.1.

Chair in Structural Engineering.

A CHAIR in Structural Engineering has been created in the Faculty of Technology of the University of Manchester and the Manchester College of Science and Technology. This is the first Chair in the subject in this country. The appointment of Mr. Wilfred Merchant, M.A., D.Sc., has been approved.

Symposium on "Shell" Roofs.

A SYMPOSIUM on shell roof construction, organised by the Norwegian Engineering Society and the Norwegian Concrete Association, is to be held at Oslo from July 1 to 3, 1957. Further details from the Organising Secretary, Den Norske Ingeniøforening, Kronprinsens gt. 17, Oslo.

Lectures on Building.

THE following lectures have been arranged by the Ministry of Works. Admission is free.

Thermal Insulation of Buildings. By Frank King. Golden Lion Hotel, King Street, Stirling. February 14, 7.15 p.m.

Safety in the Building Industry. By J. A. Hayward. College of Art, Green Lane, Derby. February 14, 7.15 p.m. And College of Technology, Anglesea Road, Portsmouth. February 20, 7.15 p.m.

The Builder and his Contract. By Norman P. Greig. Technical College, Stoke Park, Guildford. February 27, 7.15 p.m.

Recent Developments in Prestressed Concrete. By S. C. C. Bate. College of Technology, Maidstone Road, Chatham, Kent. February 14, 7.15 p.m. Also Technical College, Chesterfield Road South, Mansfield. February 28, 7.15 p.m.

The R.I.B.A. Form of Contract. By J. Stuart Daniel. Hammersmith School of Building, Lime Grove, London, W.12. February 14, 7 p.m.

Foundation Problems. By N. H. Buchi. Technical College, Northgate, Darlington. February 19, 7 p.m.

Some Notable Concrete Buildings. By R. C. Blyth. Technical College, Cherry Street, Stafford. February 19, 7.15 p.m.

Legal Obligations of Building Contractors. By John J. Clarke. College of Further Education, Newtown Road, Hereford. February 20, 7.15 p.m.

Application of Soil Mechanics to Buildings. By A. L. Little. College of Technology, Park Street, Hull. February 20, 7.15 p.m.

Soil Mechanics in the Building Industry. By M. W. Leonard. Technical College, Denzil Road, London, N.W.10. February 22, 7 p.m.

Weathering and Deterioration of Concrete Renderings. By C. Hobbs. Technical College, Collier Road, Cambridge. February 25, 7.30 p.m.

Development Trends in Building Plant. By W. R. Matthews. College of Technology and Commerce, The Lansdowne, Bournemouth. February 26, 7.30 p.m.

Mixing and Distribution of Concrete. By A. G. Stone. Marine and Technical College, St. George's Avenue, South Shields. February 26, 7 p.m.

Introduction to Site Costing for Builders. By A. E. Chittenden. Wyndham Hotel, Dunraven Place, Bridgend. February 26, 7 p.m. Also Shire Hall, Haverfordwest. February 27, 7 p.m. And Technical College, Alban Road, Llanelly. February 28, 7 p.m.

Surface Finishes of Concrete. By J. G. Wilson. Harris Institute, Corporation Street, Preston. February 27, 7 p.m.

Field Maintenance of Builders' Plant. By J. Stafford. College of Technology, Howard Street, Rotherham. February 27, 7.15 p.m.

Essentials of Good Concreting. By E. E. H. Bate. Technical College, Keighley. February 28, 7.15 p.m.

The late Arthur Wates.

We regret to announce the death, at the age of 76, of Mr. Arthur Wates, who, with his brother Edward, founded in 1902 the firm of Wates, Ltd., building and civil engineering contractors.

MISCELLANEOUS ADVERTISEMENTS.

Situations Wanted, 4d. a word : minimum, 7s. 6d. Situations Vacant, 5d. a word : minimum, 10s. Other miscellaneous advertisements, 5d. a word : 10s. minimum. Displayed advertisements, 35s. per column inch. Box number 1s. extra.

Advertisements must reach this office by the 23rd of the month preceding publication.

SITUATIONS VACANT.

SITUATIONS VACANT. Consulting structural engineers in new modern offices conveniently situated in Central Liverpool have vacancies for reinforced concrete and structural steel designer-detailers, in the salary range of £600–£800 per annum. Special opportunities exist for those interested in nuclear energy projects, and appointments in this field carry additional increments in salary. Positions offered are permanent with excellent prospects. Staff bonus and superannuation schemes are in operation. Apply in confidence, giving age and full details of experience, to Box 4321, CONCRETE AND CONSTRUCTIONAL ENGINEERING, 14 Dartmouth Street, London, S.W.1.

SITUATION VACANT. Detailer-draughtsman required for varied reinforced concrete work in sub-contractors' head office opposite Euston Station. Permanent, progressive, and superannuated post. Five-days' week. Telephone for appointment. EUS. (London) 4096.

SITUATION VACANT. Civil engineering draughtsman required for work in London office of large civil engineering contractors. Work of varied nature, with scope for initiative and advancement. Commencing salary £600–£800 depending on experience. Five-days' week and luncheon vouchers. Pension scheme operates. Box 4228, CONCRETE AND CONSTRUCTIONAL ENGINEERING, 14 Dartmouth Street, London, S.W.1.

SITUATIONS VACANT. Senior reinforced concrete designers wanted by leading reinforced concrete engineers and contractors. Must be fully conversant with Code of Practice, L.C.C. Bye-Laws, and able to design light-framed structures from estimating stage to final details. Five-days' week. Pension scheme. Progressive position. Starting salary from £900 upwards according to ability. Juniors also required, similar conditions. Write Box 4172, CONCRETE AND CONSTRUCTIONAL ENGINEERING, 14 Dartmouth Street, London, S.W.1.

SITUATIONS VACANT. Assistant structural engineers required with experience of reinforced concrete design. Experience of precast techniques advantageous. Salary according to experience and qualifications. Apply in writing to TECHNICAL MANAGER, THE SCOTTISH CONSTRUCTION CO., LTD., Sighthill Industrial Estate, Edinburgh.

SITUATION VACANT. Assistant resident engineer required for major road and bridge contract in London area. Experience of reinforced and prestressed concrete bridge construction, earthworks, and concrete roads, advantageous. Salary up to £1,200 per annum. Write, stating age, qualifications and details of experience, to Harry Brompton, Consulting Engineer, 7 Hobart Place, Westminster, S.W.1.

LONDON COUNTY COUNCIL ARCHITECT'S DEPARTMENT

Vacancies for STRUCTURAL ENGINEERS in the DISTRICT SURVEYORS' SERVICE. Duties include checking of calculations and/or supervision of works in progress. Structural knowledge essential. GRADE II (salary on scale £987 to £1184), GRADE IIIS (up to £987), and ASSISTANTS (up to £818), with starting rates according to qualifications and experience.

Particulars and application forms obtainable from THE ARCHITECT (AR/EK/DS/5), County Hall, S.E.1. (2082.)

SITUATIONS VACANT. Senior reinforced concrete designers required. Must be experienced in the design of foundations and structures. High salaries will be paid to suitable applicants. Modern semi-detached houses available shortly. Grant towards removal expenses. Pension scheme. Apply the POWER-GAS CORPORATION LTD., Stockton-on-Tees.

SITUATIONS VACANT. The Trussed Concrete Steel Co., Ltd., have vacancies in their London, Harlow, Glasgow, Birmingham, and Manchester offices for reinforced concrete designers and detailers. Five-days' week, pension scheme. Apply, giving full particulars of age, education, and previous experience, to the ENGINEERING MANAGER, Truscon House, 35-41 Lower Marsh, London, S.E.1.

SITUATIONS VACANT. TAYLOR WOODROW CONSTRUCTION, LTD., offer first-class prospects for active and experienced site civil engineers, structural design engineers, and soil mechanics geologist engineers. Apply to S.E., Ruislip Road, Southall, Middlesex.

SITUATION VACANT. Consultant requires reinforced concrete detailer with elementary design knowledge Six to eight years' drawing office experience. Good wages. Hammersmith area. Box 4410, CONCRETE AND CONSTRUCTIONAL ENGINEERING, 14 Dartmouth Street, London, S.W.1.

SITUATIONS VACANT. Reinforced concrete designers and detailers required for work on varied and interesting projects. 35-hours', 5-days' week. Half-yearly bonus. Pension scheme. Apply in writing to A. C. WILSON & PARTNERS, LTD., Design House, The Mall, Ealing, London, W.5, or telephone MR. J. CUTTER at Ealing 9843 for an appointment.

SITUATION VACANT. Quantity surveyor, age 27–30, required by consulting engineer to take charge and develop quantity surveying section. Work is mainly on heavy civil and structural projects, and duties will involve negotiations with clients and contractors. Good salary, bonus, etc. Apply in writing to MALCOLM GLOVER & PARTNERS, 276 Terminal House, London, S.W.1.

SITUATION VACANT. Resident engineer required urgently for multi-story reinforced concrete office blocks, Paddington area. Only men with experience need apply in writing to CLARKE NICHOLLS & MARCEL, Consulting Engineers, 21-29 Westbourne Grove, Bayswater, London, W.2.

SITUATIONS VACANT. Reinforced concrete detailer-draughtsmen required with some knowledge of design. Must be capable of preparing foundation layouts. Salary commensurate with experience. Apply to PILING DEPARTMENT, THE CEMENTATION CO. LTD., 20 Albert Embankment, London, S.E.11. Telephone: RELiance 7654.

THE CEMENTATION CO. LTD.

invites applications from university graduates or other suitably qualified persons for the following posts in their Research and Development Laboratories at Bentley, near Doncaster. Salary will in each case depend upon age and experience. Excellent long-term prospects are open and the positions are permanent and superannuable.

1. PHYSICAL CHEMIST—A sound knowledge of the physical and chemical properties of colloids and the crystal structure of silicate minerals is essential.

2. CONCRETE ENGINEER—A general understanding of the factors affecting the properties of cement, grout and concrete, and experience in the development and testing of these materials in the laboratory are required.

Applications in respect of both these positions giving full particulars of educational and professional qualifications, age, experience, and salary required, should be addressed to the Research Engineer, THE CEMENTATION CO. LTD., P.O. Box No. 2, Bentley Works, Doncaster.

SITUATION VACANT. Consulting engineers, Westminster, require a designer of structures able, under general direction of a partner, but otherwise by himself, to devise a structural scheme and to develop it in detail for construction by contractors, and generally to supervise its construction. Such work will be mainly for multi-story buildings. Salary to begin £1200 to £1500 according to experience. Box 4413, CONCRETE AND CONSTRUCTIONAL ENGINEERING, 14 Dartmouth Street, London, S.W.1.

SITUATION VACANT. Reinforced concrete designer-draughtsman for work in Sunbury or London office. Five-days' week. Salary £800-£1000 according to experience. Apply, stating age and details of experience, to J. H. COOMBS & PARTNERS, Consulting Engineers, Thames Corner, Sunbury-on-Thames, Middx.

SITUATIONS VACANT. Reinforced concrete designers and detailers interested in widening their experience should apply to Ove Arup & Partners, 8 Fitzroy Street, London, W.1. Telephone: Langham 7781.

SITUATIONS VACANT. R. L. BOURQUI & PARTNERS, Thermix House, Kingston-on-Thames, require detailers immediately for reinforced concrete work. Only first-class draughtsmen with good printing and linework considered. Knowledge of design an advantage, but ability to detail from calculations essential. Should be able to construct general arrangement drawings from sketches of industrial structures, framed buildings, etc. Above-average salaries offered to competent and conscientious men. Full holidays this year. Telephone for appointment to Elmbridge 0439 (office hours) or Elmbridge 2654 (evenings).

SITUATION VACANT. Senior structural engineer required by growing firm of consulting engineers. Applicants preferably over 30 years old and experienced in design and site supervision of reinforced concrete. A knowledge of steelwork would be an advantage. Varied and interesting work, good salary and excellent prospects. Offices in Central London. Five-days' week. Holiday arrangements will be honoured. Send full particulars, and salary required, in confidence, to ALAN MARSHALL & PARTNERS, 115 Gloucester Place, London, W.1.

AIR MINISTRY Works Designs Branch requires in London, STRUCTURAL ENGINEERING DESIGNER/DRAUGHTSMEN for reinforced concrete or structural steel work with sound technical training and several years' varied experience in design/detailing of:—

(a) Reinforced concrete construction for all types of buildings, or
(b) Steel framed sheds, warehouses and similar buildings.

Salaries up to £790 per annum. Starting pay dependent upon age, qualifications and experience. Long-term possibilities with promotion and pensionable prospects. Five-days' week. Three weeks three days' leave a year. Normally natural born British subjects. Write, stating age, qualifications, employment details, including type of work done, to any Employment Exchange, quoting Order No. Borough 1001.

SURREY COUNTY COUNCIL

Applications invited for following appointments:—

1. ASSISTANT STRUCTURAL ENGINEER GRADE IV, £727 15s. od. to £907 2s. 6d. p.a. plus £30 London Allowance. Qualified Civil or Structural Engineers experienced in design and detailing in steel and/or reinforced concrete. Ability to deal with a contract through all stages.
2. STRUCTURAL ENGINEERING ASSISTANT GRADE II, £609 17s. 6d. to £691 17s. 6d. p.a. plus L.A. up to £30 p.a. Experienced in detailing steelwork and/or reinforced concrete for medium sized buildings. Ability to design an advantage.

Full details, present salary and 3 copy testimonials to County Architect, County Hall, Kingston, as soon as possible.

LONDON COUNTY COUNCIL

ARCHITECT'S DEPARTMENT

Vacancies in the Structural Engineering Division for Resident Engineers (salary up to £987). Candidates should have B.Sc., A.M.I.C.E., or A.M.I.Struct.E., preferably with experience in supervision of reinforced concrete and/or steel-framed structures on the site, although those with adequate knowledge and experience of design alone will also be considered. Application forms from ARCHITECT (AR/EK/RE/4), The County Hall, London, S.E.1. (16.)

SITUATION VACANT. Reinforced concrete detailing draughtsman, with knowledge of design an advantage, required by consulting engineer engaged in all types of structures. Salary £500-£750 according to experience. Write, stating full particulars, to P. R. ROSE, A.M.I.Struct.E., 7 Connaught Place, London, W.2.

SITUATIONS VACANT. Birmingham district. Designer-detailers required for reinforced concrete specialists' office. Varied and interesting work on many different types of structures. Modern offices. Five-days' week, pension scheme, canteen. Apply 'TWISTEEL' REINFORCEMENT LTD., Alma Street, Smethwick, Staffs.

SITUATIONS VACANT. Precast prestressed concrete. Senior and junior designers and detailers required for interesting work on structural frames for all types of buildings. Good progressive salaries with profit-sharing bonus. Pension and life assurance schemes. CONCRETE LTD., Green Lane, Hounslow, Middx.

SITUATION VACANT. Company of reinforced concrete engineers require a technically trained representative to function as assistant to their divisional engineer operating from their York office. Write, stating age, experience and present salary, to Box CE.285, 191 Gresham House, London, E.C.2.

SITUATIONS VACANT. Draughtsmen required for setting-out and detailing masonry. Experience of large contracts necessary. Good salary, five-days' week, superannuation. Write GIRLINGSTONE, Feltham, Middx.

SITUATIONS VACANT. Draughtsmen experienced in precast concrete hollow-beam flooring required. Good salary, superannuation. Write GIRLINGSTONE, Feltham, Middx.

SITUATIONS VACANT. Acrow require draughtsmen and designer-draughtsmen, age around 25 years, for their steel formwork division. Vacancies occur due to existing staff being promoted to important positions in our Canadian company. Highest salaries paid, and excellent prospects of advancement afforded to competent men capable of working on own initiative. Apply, giving full details of experience, and commencing salary required, to Mr. R. J. TOOLEY, ACROW (ENGINEERS) LTD., South Wharf, Paddington, London, W.2.

SITUATIONS VACANT. Consulting engineers require reinforced concrete designers and detailers for varied and interesting structures. Opportunity for experience of steelwork. Five-days' week. Applications in strict confidence stating full details, including salary required, to ALAN MARSHALL & PARTNERS, 115 Gloucester Place, London, W.1.

LONDON COUNTY COUNCIL

ARCHITECT'S DEPARTMENT

Vacancies for Engineering Assistants (up to £817) and Engineer Grade III (up to £987) in the Structural Engineering Division.

The Division's work covers a programme of total value £12 millions this year, including multi-story flats, schools, office, warehouse and other buildings.

Particulars and application forms from the ARCHITECT (AR/EK/SE/3), The County Hall, London, S.E.1. (15.)

POSTS OF CHIEF ENGINEER AND POWER ENGINEER, CEMENT FACTORY, KANKESANTURAI, CEYLON.

Applications are invited for the posts of:

(a) Chief Engineer

(b) Power Engineer

for the Cement Factory, Kankesanturai, Ceylon.

(a) CHIEF ENGINEER.—Emoluments: For non-Ceylonese, between Rs. 2,100-100-3,000 per month without allowances but free of income tax.

QUALIFICATIONS: (1) Degree in mechanical engineering and/or corporate membership of a recognized professional institution of mechanical engineers.

(2) (a) Sound practical experience as a mechanical engineer in a responsible capacity.

(b) Not less than 5 years' experience as a cement works' engineer with specialized knowledge of the dry process of cement manufacture.

(3) Experience in power generation, distribution and installation. A knowledge of chemical engineering will be an added qualification.

(4) Knowledge of the English language.

(b) POWER ENGINEER.—Emoluments: For non-Ceylonese, between Rs. 1,800-100-2,200 per month without allowances but free of income tax.

QUALIFICATIONS: (1) Corporate membership of a recognized professional institution of electrical engineers.

(2) Sound practical experience in a responsible capacity in the following:

(a) operation and maintenance of high powered Diesel engines.

(b) power generation, transmission and installation.

(3) Knowledge and experience of mechanical engineering will be an added qualification.

(4) Knowledge of English language.

Terms of Engagement: The appointment to each post will be on agreement for a period of 3 years in the first instance subject to renewal by mutual consent.

Note: The selected candidate for each of the above posts may be placed at a point on the scale at the discretion of the Corporation Board having regard to his qualifications and experience.

Applications should reach the undersigned not later than 20 February, 1957.

Further particulars to be obtained from the office of the Ceylon Embassy or the Ceylon High Commission for the countries concerned.

C. COOMARASWAMY,
Chairman,
"Kankesan Cement Works" Corporation.
110 General's Lake Road,
Colombo, 2, Ceylon.

31 December, 1956.

SITUATION VACANT. Senior designer-draughtsman required by well-known London firm of reinforced concrete engineers. Applicants must be experienced in the design of reinforced concrete structures as applied to housing and commercial buildings, and familiar with Codes of Practice and Building Regulations. Good salary offered. Progressive post. Pension scheme. Write fullest particulars to Box 4415, Concrete and Constructional Engineering, 14 Dartmouth Street, London, S.W.1.

THE PORT OF LONDON AUTHORITY

have vacancies in their Drawing Offices for Technical Assistants of various grades, for Structural Engineering Design and Detailing. Salary within the range of £645 to £956 per annum.

Minimum qualifications are the holding of the Ordinary National Certificate in appropriate subjects and at least 3 years' experience in a design drawing office. Higher National Certificate is required for the higher grades. Experience in the design and detailing of reinforced concrete is essential but knowledge of steelwork design will be an advantage.

It is the Authority's policy to encourage Technical Assistants to study for higher qualifications and facilities are provided accordingly.

Applications giving age, qualifications, and full details of experience should be addressed to the Establishment Officer, Port of London Authority, Trinity Square, London, E.C.3.

SITUATION WANTED.

SITUATION WANTED. Part-time work wanted. Experienced engineer, B.Sc., offers to assist consultants by undertaking structural design work, including calculations and drawings. Box 4414, Concrete and Constructional Engineering, 14 Dartmouth Street, London, S.W.1.

FOR SALE.

FOR SALE. New bar-benders for immediate delivery. Besco No. 2 size hand-lever operation. Capacity: Cold bars up to 4 in. × 5/16 in.; 7/8 in. diameter; 3/4 in. square; notched angles 4 in. × 5/16 in. Hot bars up to 4 in. × 5/8 in. or equivalent. Special blocks available. Fully illustrated leaflet and price from F. J. Edwards Ltd., 359 Euston Road, London, N.W.1 (Euston 4681), or 41 Water Street, Birmingham, 3 (Central 7606).

FOR SALE. "Scriven" motor-driven punching and angle shearing machine. Punches up to 1 in. diameter through 3/4 in. thick. Shears angles 6 in. × 6 in. × 3/4 in. Depth of gap 13 3/4 in. With or without electrical equipment. Weight about 16 1/2 tons. Photos and full details from F. J. Edwards Ltd., 359 Euston Road, London, N.W.1 (Euston 4681), or 41 Water Street, Birmingham, 3 (Central 7606).

FOR SALE. New Schluter "baby" shearing machines for sale. Suitable for cutting plates of any length or width, 5/16 in. thick or equal. Arranged for motor drive, 400 volts, 3-phase, 50 cycles. Length of blades 17 3/4 in. Capacity: round bars 3/4 in., squares 5/8 in. Others with 12 1/2-in. blades for 5/16-in. plates, and 9 3/4-in. blades for rounds and squares up to 3/4 in., plates, angles, tees, etc. Full details available from F. J. Edwards Ltd., 359 Euston Road, London, N.W.1.

FOR SALE. Wiedemann type R.4 hand-operated turret punching machine with twelve stations. Steel plate construction. Fitted with punches and dies from 1/4 in. to 1 1/4 in. diameter. Capacity 5/8 in. diameter holes through 5/16 in. thick, or 1 1/4 in. diameter through 14 s.w.g. Depth of gap to frame, 24 in. Photo, etc., from F. J. Edwards Ltd., 359 Euston Road, London, N.W.1.

FOR HIRE.

FOR HIRE. Lattice steel erection masts (light and heavy), 30 ft. to 150 ft. high, for immediate hire. Bellman's, 21 Hobart House, Grosvenor Place, London, S.W.1.

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